Efficient Blind Equalizer Schemes Using Variable Tap-length Algorithm

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Abstract: InterSymbol interference (ISI) distortion can be rectified without training sequence using blind equalization schemes. However, such a capability of the equalization methodology comes at the expense of high cost, and therefore it is necessary for the designers to think about efficient schemes to accomplish the blind equalization process. In this work, using a variable tap-length strategy, two algorithms for equalizing quadrature amplitude modulation (QAM) signals are proposed and tested. generalized Sato algorithm (GSA) and constant modulus algorithm (CMA) are incorporated with a variable tap-length technique to update the filter's coefficients. The variable tap-length method utilizes a fractional filter length in every iteration to optimize the filter coefficients and structure. Simulations are conducted in various channels for 16-QAM and 64-QAM, and the results have shown a considerable improvement in both mean square error (MSE) characteristics of the suggested algorithms as well as the ability of the presented algorithms to search for the optimal length.

Keywords: Blind Equalization Algorithms, Variable Tap-length Algorithm, Adaptive Filters, Constant Modulus Algorithm, QAM Signals.

1. Introduction

In communication systems adaptive equalization process is used to compensate for signal distortion incurred by the convolution between the transmitted signal and the channel. When the statistics of the transmitted signal utilised to infer the signal at the receiver side, this is called blind equalization [1]. Large number of blind equalization algorithms are introduced in the literature [2-4]. General Sato algorithm [5], constant modulus algorithm [6] and multimodulas algorithm (MMA) [7], to name a few, are just examples of well-known blind equalizers. In QAM receivers, the equalizers normally consume a substantial portion of the demodulation process [8], which necessitates the need to design cost-effective blind equalization algorithms. In [9] the authors used digital watermarking technique to improve the computational complexity of the equalizer, however, such a method requires a payload to be carried in order for the algorithm to accomplish the equalization process. The authors of [10] on the other hand, introduced a structure criterion switching control, by switching between adaptation modes and they utilised this strategy for blind decision feedback equalizer (DFE). A new CMA equalizer with optimum tap-length for QAM signals was developed in [11] and the proposed algorithm was compared with different fixed length CMA equalizers. In this paper, a variable tap-length called method called fractional tap-length (FT) algorithm [12], is combined with two blind equalizers, namely, GSA and CMA to optimise the filters' coefficients and size at the same time. This work is organized as the following: in Section 2 both of the GSA and CMA blind equalizers are described; in Section 3, the optimal tap-length algorithm is presented; in Section 4, the two proposed fractional tap-length equalizers will be presented, that is, the fractional tap-length GSA (FT-GSA) and the fractional tap-length CMA (FT-CMA); the system model and the simulation results are demonstrated in Section 5 within the context of single input single output (SISO) setup; and finally Section 6 will conclude the paper.

2. Blind Equalization Algorithms

The LMS equalizer tap update algorithm is defined by as:

$$\boldsymbol{w}(n+1) == \boldsymbol{w}(n) + \mu e(n)\boldsymbol{x}^*(n) \qquad (1)$$

Where $e(n)$ is the error signal for a particular algorithm μ is a constant step size and $x^*(n)$ is the complex conjugation of the input signal vector. Two distinguished error signals are defined in this paper for two blind equalizers.

2.1. General Sato Algorithm

A stochastic gradient-descent equalizer adjustment algorithm can be written as:

$$\boldsymbol{w}(n+1) = \mu(\gamma_s\, sgn(y(n)) - y(n))\boldsymbol{x}^*(n) \quad (2)$$

Where γ_s is a constant of the QAM constellation, $y(n)$ is the equalizer output and $sgn(.)$ is the real valued sign operator and the error signal for GSA algorithm can be defined according to the following

$$w(n+1) = \mu(\gamma_s\, sgn(y(n)) - y(n))x^*(n) \quad (3)$$

$$\boldsymbol{w}(n+1) = \mu(\gamma_s\, sgn\big(y_R(n)\big) - y_R(n) + j(\gamma_s\, sgn\big(y_I(n)\big) - y_I(n))\boldsymbol{x}^*(n) \qquad (4)$$

And the error signal for GSA is given by:

$$e^{gsa}(n) = (\gamma_s\, sgn\big(y_R(n)\big) - y_R(n) + j(\gamma_s\, sgn\big(y_I(n)\big) - y_I(n)) \qquad (5)$$

2.2. Constant Modulus Algorithm

The CMA equalizer's coefficient update is given by:

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu(y_R(n)\big(\gamma_C^2 - y_R^2(n) - y_I^2(n)\big) + jy_I(n)\big(\gamma_C^2 - y_R^2(n) - y_I^2(n)\big))\boldsymbol{x}^*(n) \quad (6)$$

Where γ_C^2 is the dispersion constant and is given in [] by:

$$\gamma_C^2 = \frac{E[|s(n)|^4]}{E[|s(n)|^2]} \qquad (7)$$

Here, the $s(n)$ is the source signal and hence the CMA error signal can by written as:

$$e^{cma}(n) = e_R^{cma}(n) + je_j^{cma}(n) \qquad (8)$$

$$e_R^{cma}(n) = y_R(n)\big(\gamma_C^2 - y_R^2(n) - y_I^2(n)\big) \qquad (9)$$

$$e_I^{cma}(n) = y_I(n)\big(\gamma_C^2 - y_R^2(n) - y_I^2(n)\big) \qquad (10)$$

3. Optimal Tap-length

Using variable tap-length algorithm, the adaptive filter's weight update within a system identification model is given by:

$$\boldsymbol{w}_{L(n)}(n+1) = \boldsymbol{w}_{L(n)}(n) + \mu e_{L(n)}^{(L(n))} \boldsymbol{X}_{L(n)}(n) \qquad (11)$$

Here, $w_{L(n)}$ and $X_{L(n)}$ are the coefficient update and input signal respectively, μ is the step size, $L(n)$is the variable filter's tap-length and $e_{L(n)}^{(L(n))}$ is defined by [12] to be the segmented error and calculated using the equation.

$$e_S^{(L(n))}(n) = d(n) - \boldsymbol{w}_{L(n);1:S}^T(n)\boldsymbol{X}_{L(n);1:S}(n) \qquad (12)$$

Where, $1 \leq S \leq L(n)$, $d(n)$ is the desired signal and $w_{L(n);1:S}(n)$ and $X_{L(n);1:S}(n)$ are vectors of first S elements of the filter's weight and input samples respectively. A pseudo fractional tap-length therefore, can defined as the following:

$$l_f(n+1) = (l_f(n) - \alpha) - \gamma[((e_{L(n)}^{(L(n))}(n))^2 - ((e_{L(n)-\Delta}^{(L(n))}(n))^2] \qquad (13)$$

Where α is some positive leakage parameter, Δ is a positive integer and γ is the step for tap-length update process. The integer tap-length update for the next iteration is found using the fractional tap-length by the following:

$$L(n) = \begin{cases} \lfloor l_f \rfloor & if \ \ |L(n) - l_f| > \delta \\ L(n) & \qquad otherwise \end{cases} \qquad (14)$$

And here, δ is a small integer.

The fractional tap-length algorithm described in the previous section is incorporated in GSA and CMA blind equalizes' weight update to achieve new blind equalizers that are capable of searching for the optimal structure while the filters are adapting and as such, the GSA weight update can be rewritten as:

$$\boldsymbol{w}(n+1)_{L(n)} = \mu(\gamma_s \, sgn(y(n)) - y(n))_{L(n)} \boldsymbol{x}^*(n)_{L(n)} \qquad (15)$$

Where,

$$e^{gsa}(n) = (\gamma_s \, sgn\big(y_R(n)\big) - y_R(n) + j(\gamma_s \, sgn\big(y_I(n)\big) - y_I(n)) \qquad (16)$$

The CMA weight update can be rewritten as

$$w(n+1)_{L(n)} = w(n)_{L(n)} + \mu |e^{cma}(n)|_{L(n)} x^*{}_{L(n)}(n) \qquad (17)$$

Where, $$e_{L(n)}^{(L(n))} = |e_R^{cma}(n) + je_j^{cma}(n)|_{L(n)} \qquad (18)$$

And, $$e_{L(n)-\Delta}^{(L(n))} = |e_R^{cma}(n) + je_j^{cma}(n)|_{L(n)-\Delta} \qquad (19)$$

4. System modelling and simulations:

A single input single output (SISO) system model illustrated below in Fig.1 is used for the simulation. In this model $'n'$ is the T-spaced and $'k'$ is $T/2$-spaced quatities. The source transmits symbols which are independent and identically distributed random variables. The equalizer's input vector samples is given by

$$\boldsymbol{X}_{L(n)}(n) = \boldsymbol{C}_{L(n)}^T \boldsymbol{S}_{L(n)}(n) + \boldsymbol{v}_{L(n)}(n) \qquad (20)$$

Where, $C_{L(n)}$ is variable length T-spaced convolution matrix [14], $v_{L(n)}(n) = [v_1(n), v_2(n), \ldots . v_{L(n)}(n)]^T$ is the Gaussian noise of length $L(n)$, and hence, the equalizer output is decimated by a factor of two and is given by;

$$y_{L(n)}(n) = \boldsymbol{X}_{L(n)}^T(n)\boldsymbol{w}_{L(n)}(n) = \boldsymbol{S}_{L(n)}^T(n)\boldsymbol{C}_{L(n)}(n)\boldsymbol{w}_{L(n)}(n) + \boldsymbol{v}_{L(n)}^T \boldsymbol{w}_{L(n)}(n) \qquad (21)$$

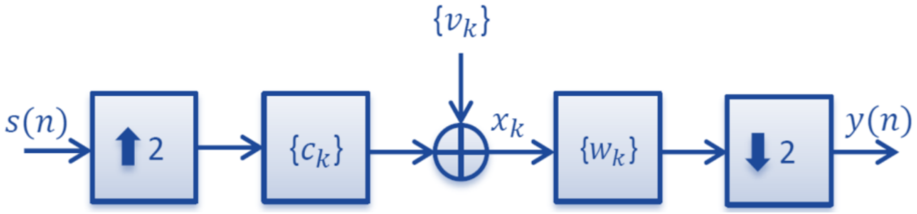

Figure 1. A single input single output – SISO system model.

The two proposed algorithms (FT-GSA) and (FT-CMA) in Section 3.1 are tested in an experimental setup that involves a $T/2$-spaced SPIB microwave channel [13] and a $T/2$-spaced equalizer's impulse response which is initialized by a unitary double centre spike. The received symbols at the equalizer are obtained by convolving the source symbols and the channel$SPIB\#5$. The two proposed algorithms are examined using random source symbols taken from$16-QAM$. The simulation is carried out by adding a white Gaussian noise such that the signal to noise ratio (SNR) is $30dB$. Parameters selections for both algorithms are illustrated below in Table 1.

Table 1: FT-GSA & FT-CMA parameters selections.

Parameter	FT-GSA Algorithm	FT-CMA Algorithm
$\boldsymbol{\mu}$	2^{-12}	2^{-12}
Δ	3	3
$\boldsymbol{\gamma}$	0.0025	0.0025
$\boldsymbol{\alpha}$	0.00065	0.00065
$\boldsymbol{\delta}$	1	1

System simulation was implemented by averaging 100 independent experiments, and the mean square error (MSE) of both proposed algorithms were averaged and compared with fixed length blind equalizers algorithm, that is, GSA, CMA and MMA, all with fixed tap-length of 18 taps. The MSE error properties of the proposed algorithms against fixed length equalizers are shown in Figure 2.

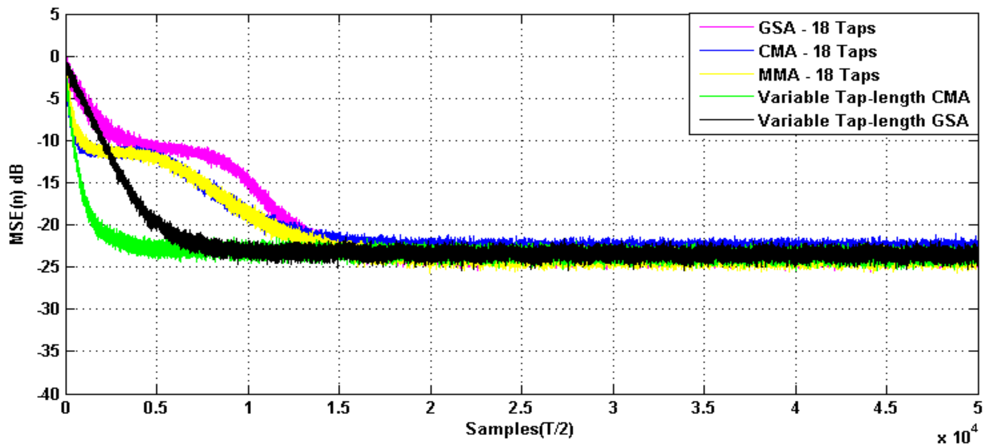

Figure 2. Simulation results of proposed algorithms against other fixed length equalizers.

It is obvious from Figure 2 that, the proposed algorithms have sown better mean square error and convergence rate than other fixed length filters. The output signal constellation for 16-QAM for both, FT-GSA algorithm and FT-CMA algorithm, are shown in Figure 3 (FT-GSA in blue and FT-CMA in pink) respectively, where detectable constellation points resulted from the proposed algorithms.

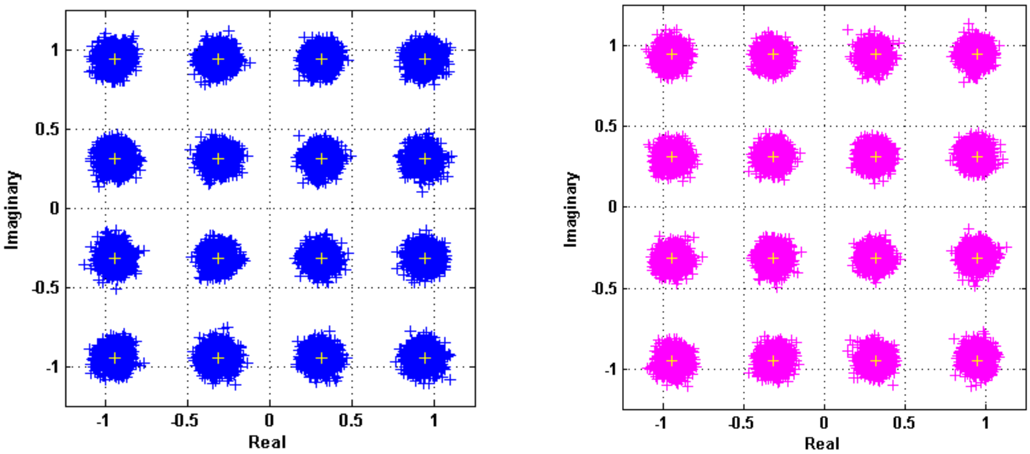

Figure 3 Output signal constellation of FT-GSA and FT-CMA for 16-QAM.

The proposed algorithms have shown the capability to search for optimal tap-length, and hence, the optimal structures for both algorithms. Figure 4 shows the expected value of tap-lengths for the FT-GSA and FT-CMA algorithms respectively.

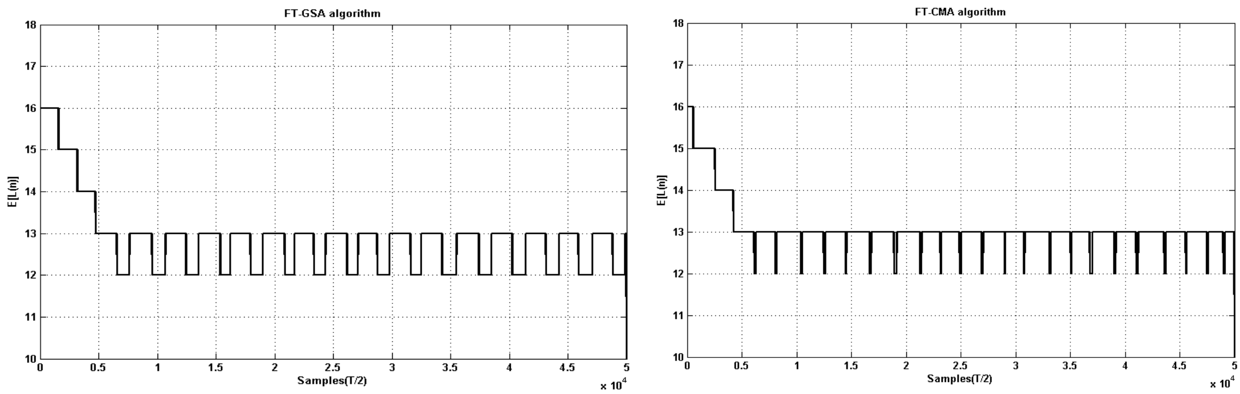

Figure 4. The expected value of tap-lengths for FT-GSA and FT-CMA.

The tap-length expected value$E[L]$ have converged to an average value of approximately $L = 12$. Thus, an approximation optimal tap-length of $L_{opt} = 12 - \Delta = 9$ taps is estimated to both algorithms.

5. Conclusion

The study has addressed InterSymbol interference distortion in communication systems through efficient blind equalization schemes. Blind equalization eliminates the need for training sequences but often incurs high computational costs, prompting the need for innovative approaches. Two tap-length algorithms were developed to dynamically optimise filter coefficients and structures using a variable tap-length method. The proposed algorithms were tested with 16-QAM signals transmitted over a T/2 spaced SPIB microwave channel. Comparisons with fixed-length blind equalizers, such as GSA, CMA, and MMA with an 18-tap configuration, proved that FT-GSA and FT-CMA achieved superior mean square error – MSE performance and faster convergence rates. The output signal constellations further confirmed the accuracy of the proposed algorithms in maintaining well-defined constellation points. Indeed, the algorithms revealed the capability to dynamically search for optimal tap-lengths, with an estimated optimal value of approximately 9 taps. This adaptability enhances computational efficiency while ensuring effective equalization. The findings establish the efficacy of FT-GSA and FT-CMA in addressing InterSymbol interference while reducing resource requirements. Future work could extend these methods to other modulation schemes and channel conditions toward further validate to their versatility and practical relevance.

6. References

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