Response Prediction
Soil-Structure Interaction (SSI)

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Outline

- **Introduction**
  - Engineering Research
  - Response Prediction
- **Response Prediction**
  - Simplified Modelling of Structures
  - Simplified Modelling of Piles:
    - *Buckling*
    - *Vibration*
    - *Scour Effect*
  - Detailed Modelling of Soil-Structure Interaction (SSI)
    - *Coupled SSI Modelling Formulations*
    - *Coupled SSI Simulation Environment*
    - *SSI Applications*
- **Future Work**
- **Concluding Remarks**

Introduction

❖ Engineering Research to …

- ❑ Explore some extremely complex phenomena in the field of Engineering
- ❑ Study those in a rigorous way

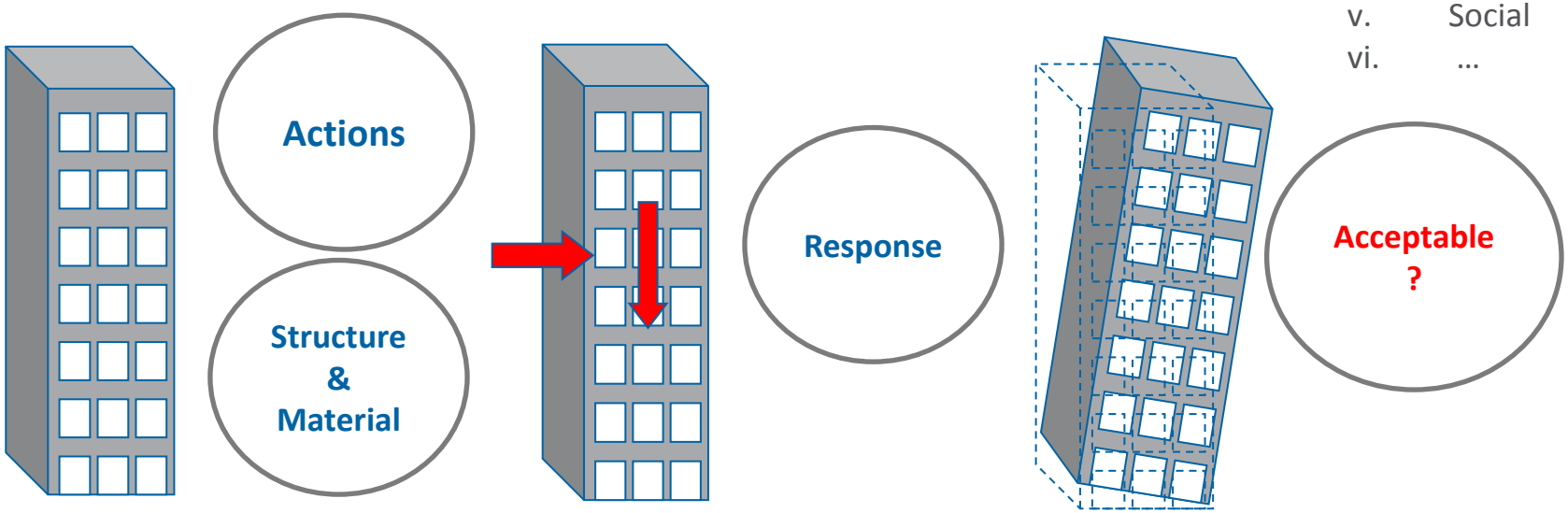
& just as importantly

- ❑ look for ways to communicate the results:
  - ➢ Aid on understanding the subject area
  - ➢ Ensure the outcomes would be of assistance in the practice

Introduction

- **Structural Engineering Research : Ensuring Predictable is Acceptable**

i. Safety
ii. Applicability
iii. Cost
iv. Environmental
v. Social
vi. …



Introduction

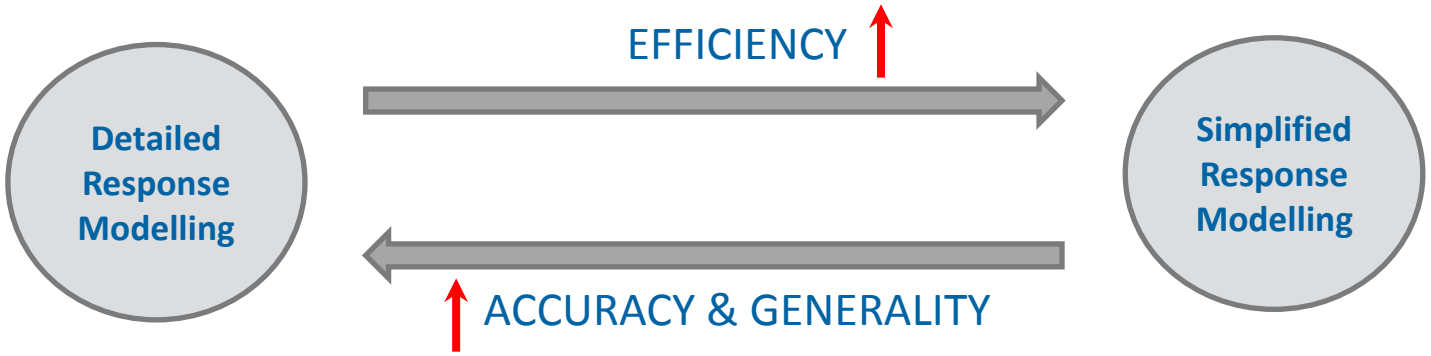
❖ Acceptability of the Response

Italy, 2009

Introduction

❖ Response Prediction

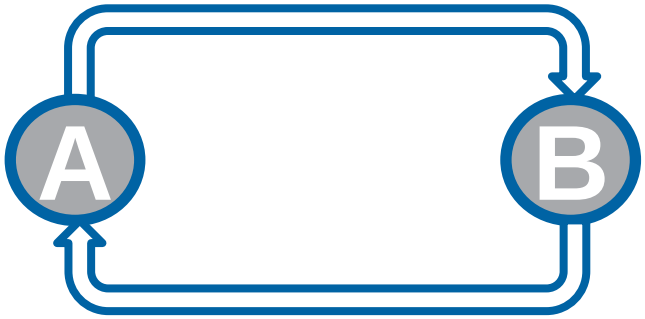
- ➢ Simplified
- ➢ Detailed



Introduction

❖ Soil-Structure Interaction

Coupled Systems:



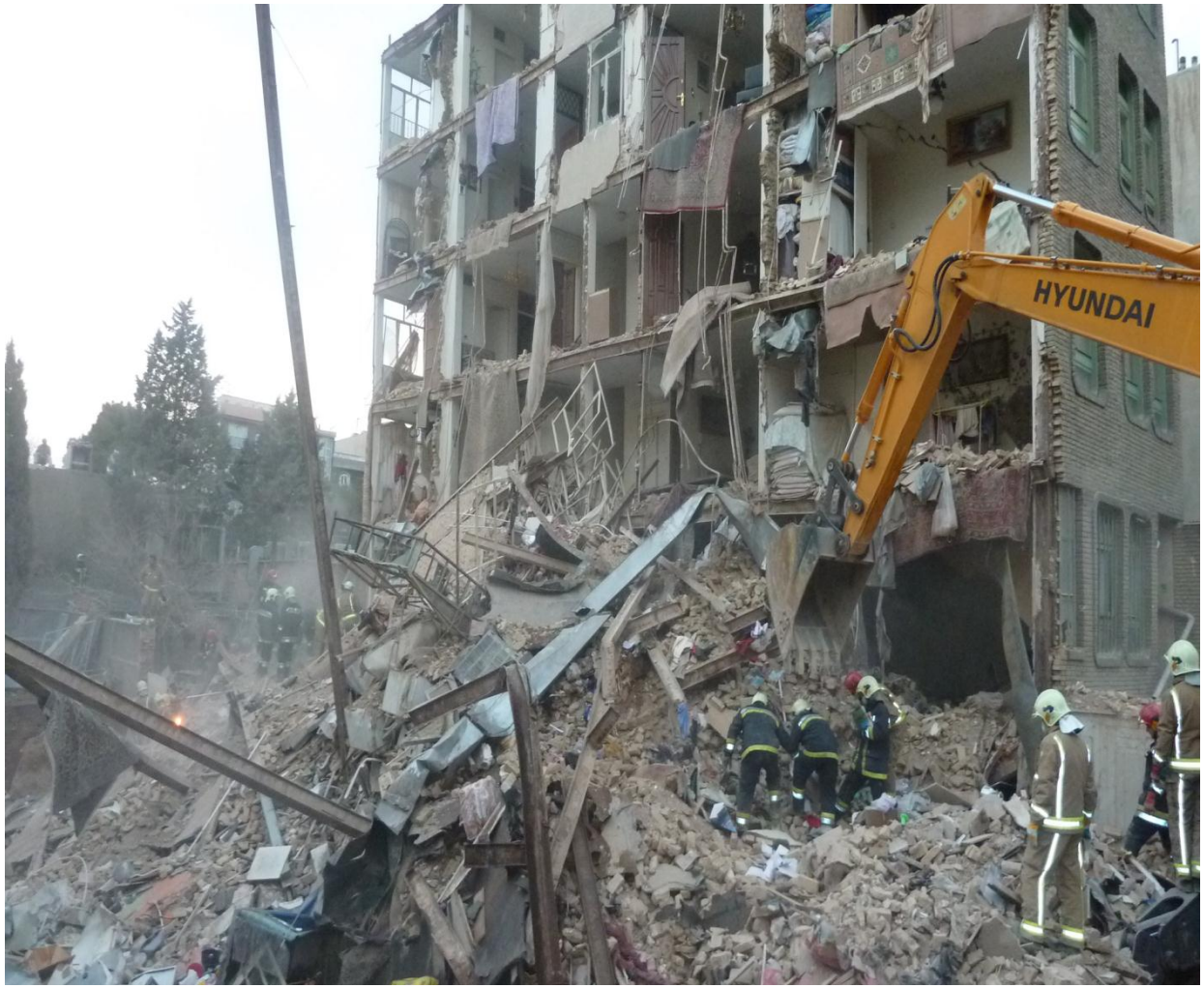
- ✓ Systems driven by the interaction of functionally distinct components (sub-domains)
- ✓ Due to interaction, the sub-domains cannot be solved separately from the other

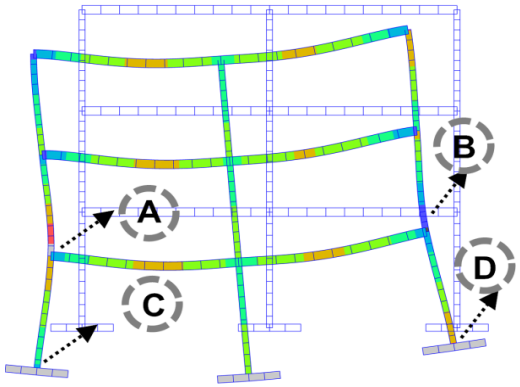
Coupled Systems

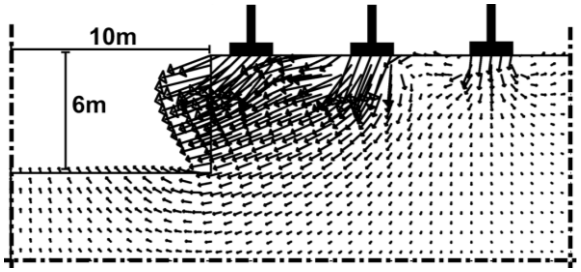
Soil-Structure / Fluid-Structure / Soil-Fluid-Structure Interaction

Coupled Systems

Soil-Structure / Fluid-Structure / Soil-Fluid-Structure Interaction







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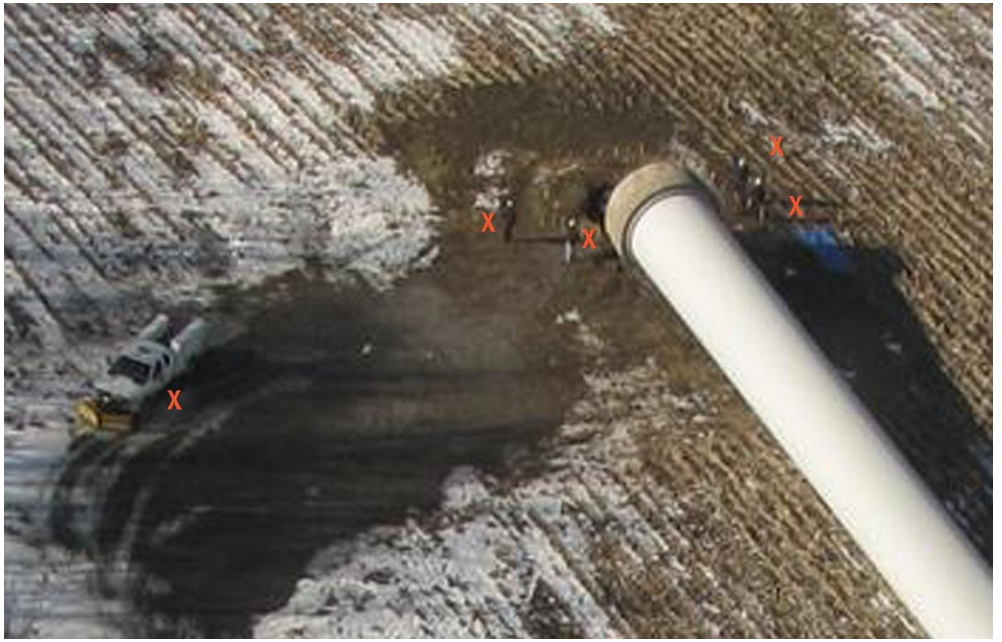

http://betterplan.squarespace.com

http://ontario-wind-resistance.org

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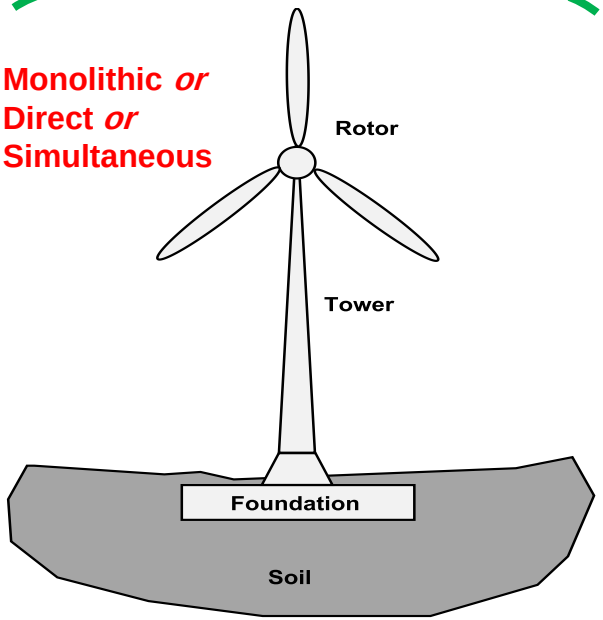
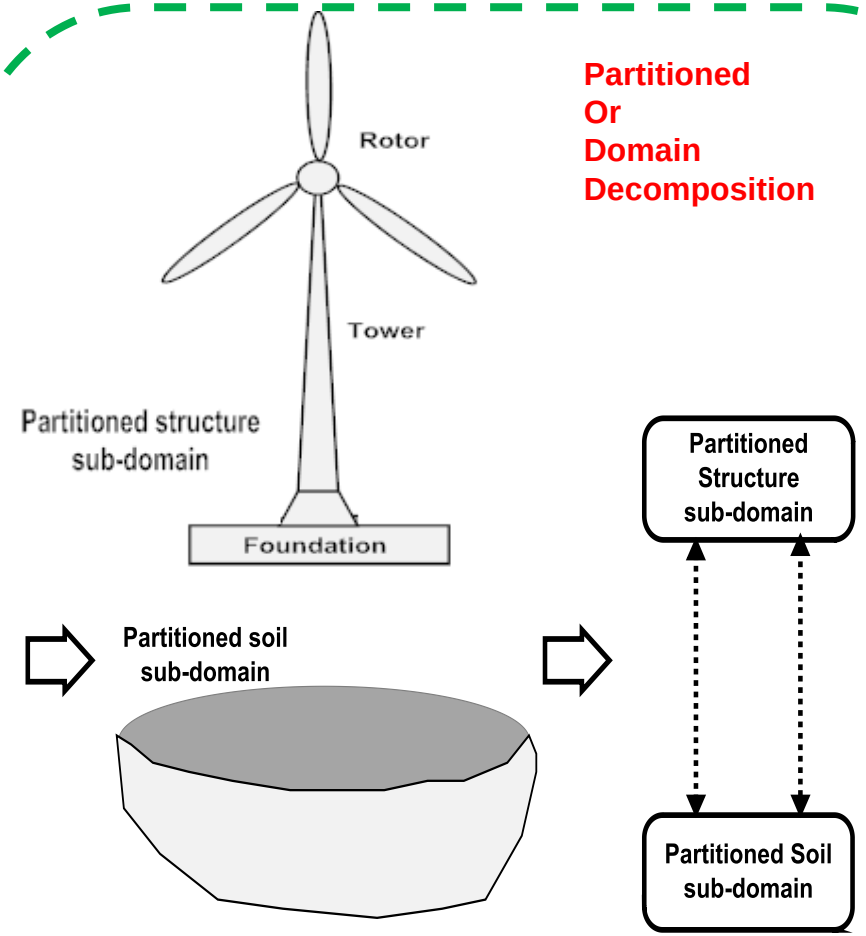
Response Prediction

❖ Coupled Soil-Structure Interaction Response Modelling:

Monolithic *or*
Direct *or*
Simultaneous



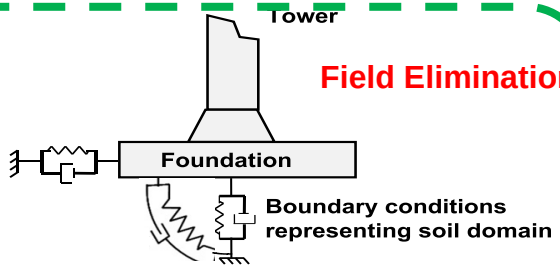
Partitioned
Or
Domain
Decomposition



b) The soil-structure coupled system is physically decomposed to separately modelled soil and structure sub-domains.

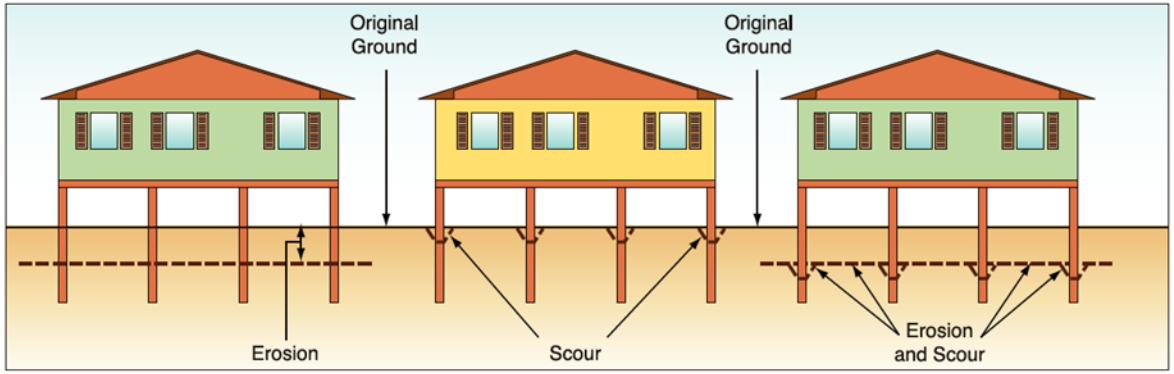
c) Coupling is achieved through exchange of the interaction effects at the interface

Field Elimination

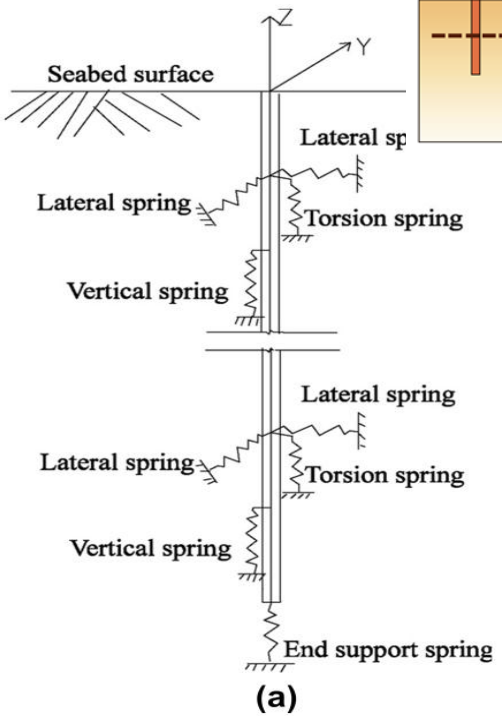


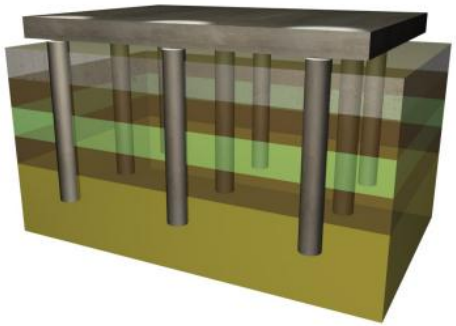
b) Idealised model for soil-structure interaction analysis using filed elimination technique

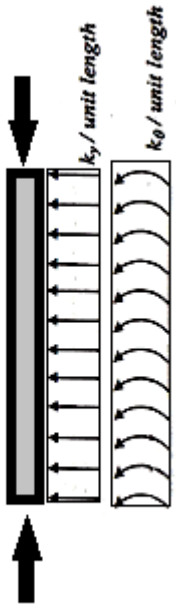
Response Prediction – Simplified Modelling

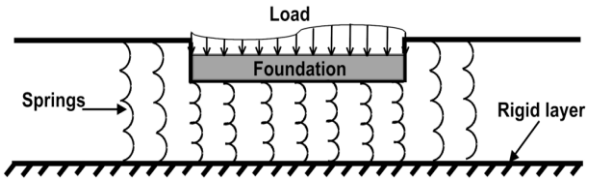


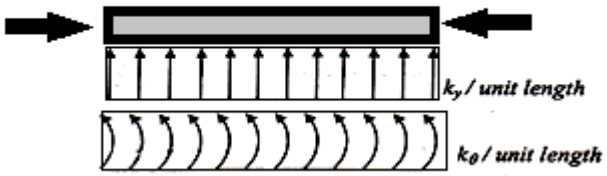
(Federal Emergency Management Agency, 2009),











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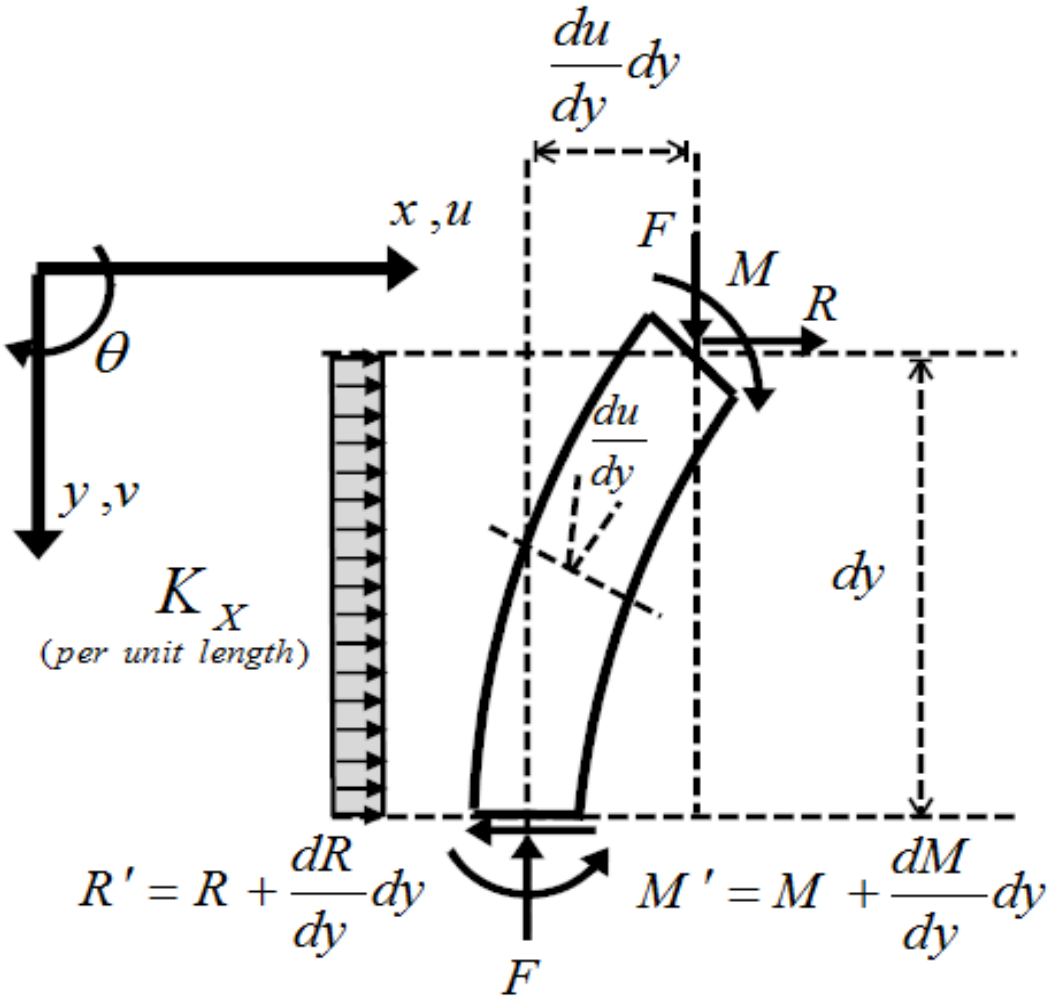
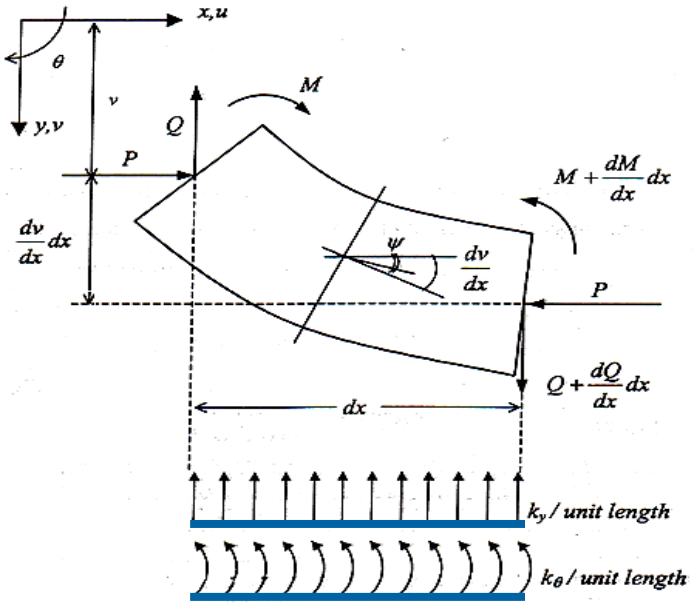
Response Prediction – Simplified Modelling

- **Vibration and Buckling of Piles on lateral and rotational support**



Response Prediction – Simplified Modelling

❖ Vibration and Buckling of beams on lateral and rotational support



$$-EI\frac{\partial^4 v}{\partial x^4}+(K_\theta-P)\frac{\partial^2 v}{\partial x^2}-K_Y.v-\rho.A.\frac{\partial^2 v}{\partial t^2}=0$$

$$V(x,t)=V(x)\sin\omega t$$

$$\frac{\partial^4 v}{\partial x^4}-\frac{(K_\theta-P)}{EI}\frac{\partial^2 v}{\partial x^2}+\frac{(K_Y-\rho A\omega^2)}{EI}v=0$$

$$\frac{\partial^4 v}{\partial \xi^4}-(K_\theta^*-P^2)\frac{\partial^2 v}{\partial \xi^2}+(K_Y^*-b^2).v=0$$

$$K_Y^*=\frac{K_Y.L^4}{EI}\qquad K_\theta^*=\frac{K_\theta.L^2}{EI}\qquad P^2=\frac{P.L^2}{EI}\qquad b^2=\frac{\rho.A.\omega^2.L^4}{EI}$$

$$v(\xi)=D_1.\sinh\alpha\xi+D_2.\cosh\alpha\xi+D_3.\sinh\beta\xi+D_4.\cosh\beta\xi$$

$$\begin{cases}\alpha^2=b^*+(b^{*2}-C)^{0.5}\\ \beta^2=b^*-(b^{*2}-C)^{0.5}\end{cases}\qquad C=K_Y^*-b^2$$

$$b^*=(K_\theta^*-P^2)/2$$

Response Prediction – Simplified Modelling

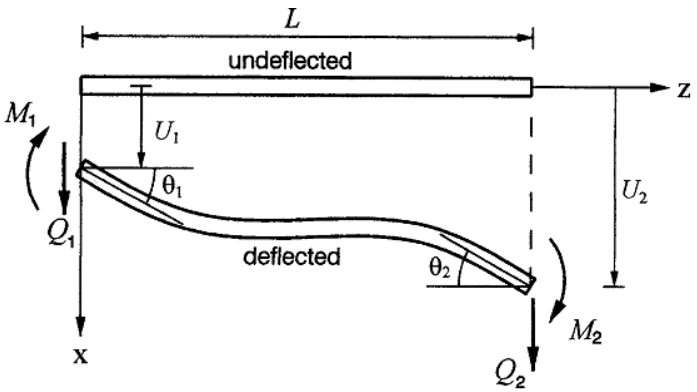
❖ Vibration and Buckling of beams on lateral and rotational support

$$V(\xi) = D_1.\cosh\alpha\xi + D_2.\sinh\alpha\xi + D_3.\sin\alpha\xi + D_4.\cos\alpha\xi$$

$$Q = \frac{-EI}{L^3}\left\{C_1.\alpha.\beta^2.\sinh\alpha\xi + C_2.\alpha.\beta^2.\cosh\alpha\xi + C_3.\beta.\alpha^2.\sin\beta\xi - C_4.\beta.\alpha^2.\cos\beta\xi\right\}$$

$$M = \frac{-EI}{L^2}\left\{D_1.\alpha^2.\cosh\alpha\xi + D_2.\alpha^2.\sinh\alpha\xi - D_3.\beta^2.\cos\beta\xi - D_4.\beta^2.\sin\beta\xi\right\}$$

$$\theta = \frac{1}{L}\left\{D_1.\alpha.\cosh\alpha\xi + D_2.\alpha.\sinh\alpha\xi + D_3.\beta.\cosh\beta\xi + D_4.\beta.\sinh\beta\xi\right\}$$



At end 1 $(\xi = 0)$ $\Rightarrow v_1 = v$ and $\theta_1 = \theta$

At end 2: $(\xi = 1)$ $\Rightarrow v_2 = v$ and $\theta_2 = \theta$

and

At end 1 $(\xi = 0)$ $\Rightarrow Q_1 = -Q$ and $M_1 = M$

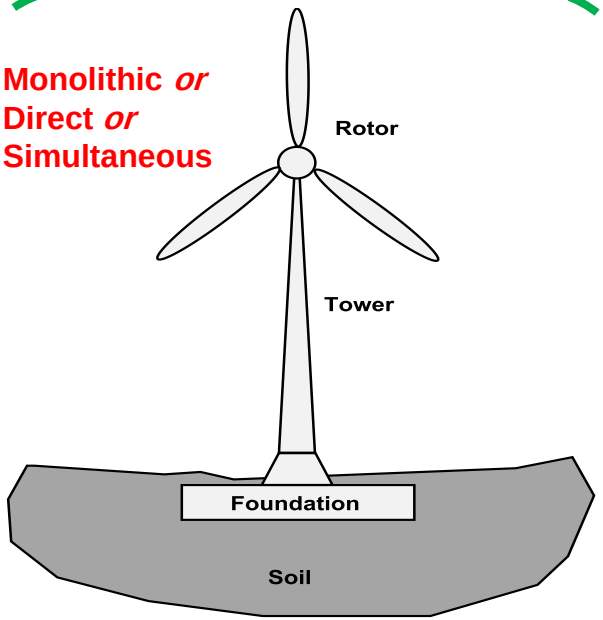
At end 2: $(\xi = 1)$ $\Rightarrow Q_2 = Q$ and $M_2 = -M$

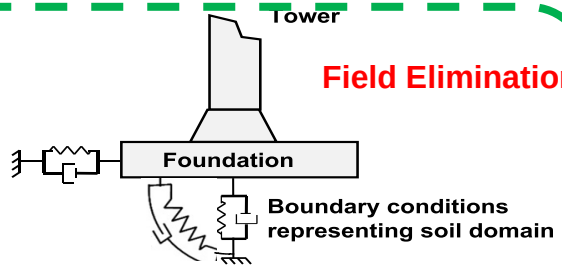
$$\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ & K_{22} & K_{23} & K_{24} \\ & & K_{33} & K_{35} \\ sym. & & & K_{44} \end{bmatrix} * \begin{bmatrix} V_1 \\ \theta_1 \\ V_2 \\ \theta_2 \end{bmatrix}$$

Detailed Response Modelling

❖ Coupled Soil-Structure Interaction Response Modelling:

Monolithic *or*
Direct *or*
Simultaneous





Field Elimination

b) Idealised model for soil-structure interaction analysis using filed elimination technique

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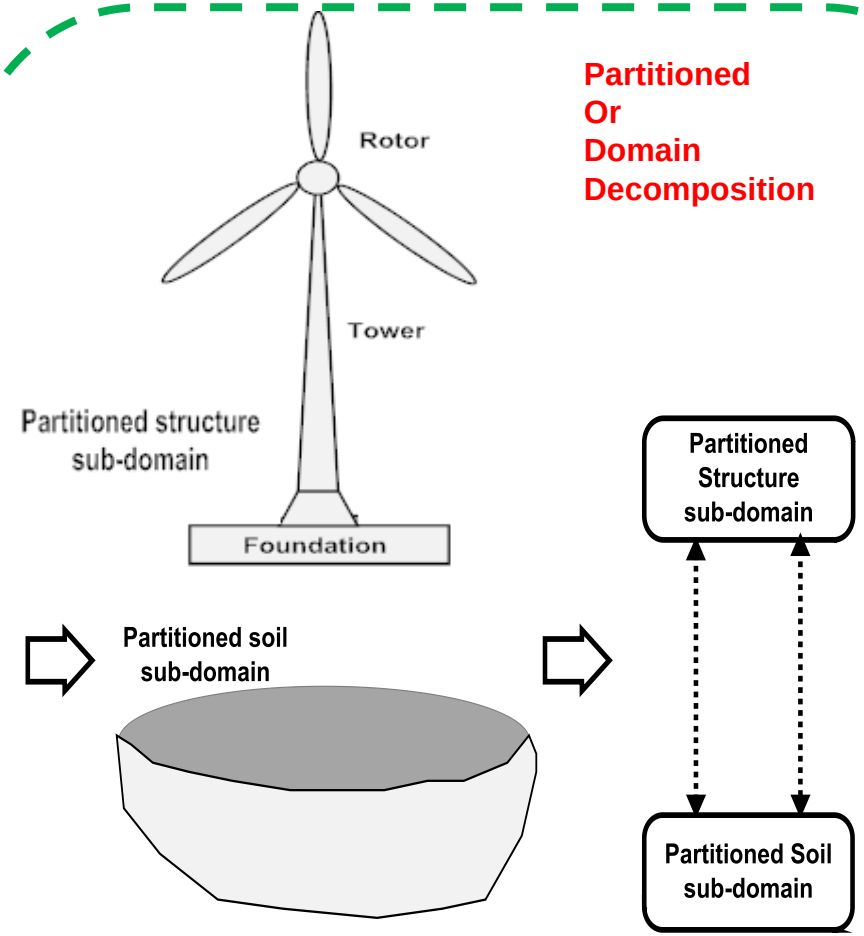

b) The soil-structure coupled system is physically decomposed to separately modelled soil and structure sub-domains.

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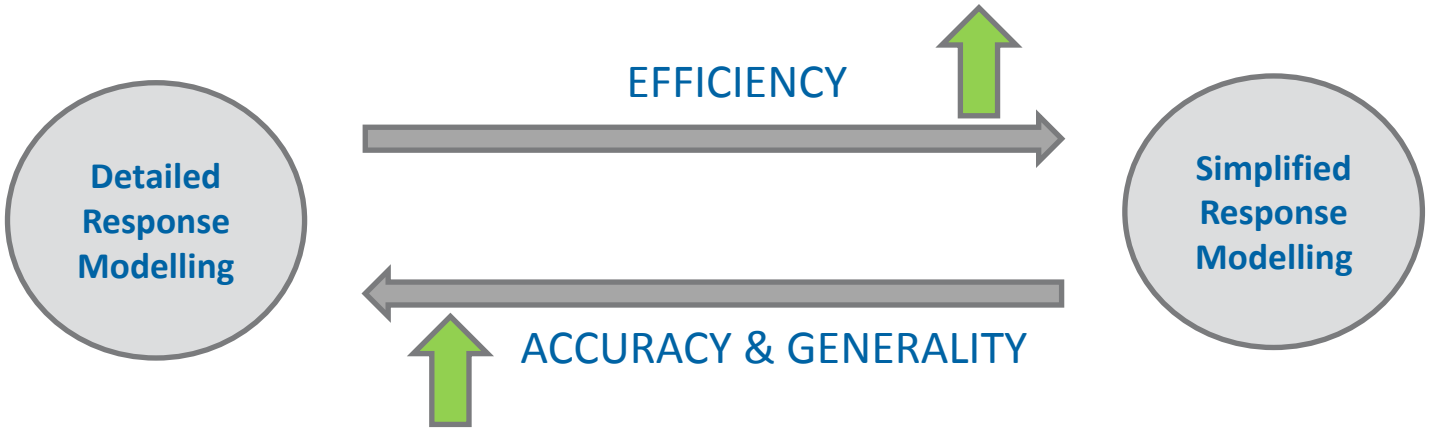
Detailed Response Modelling

- **Coupled Soil-Structure Interaction Response Modelling:**

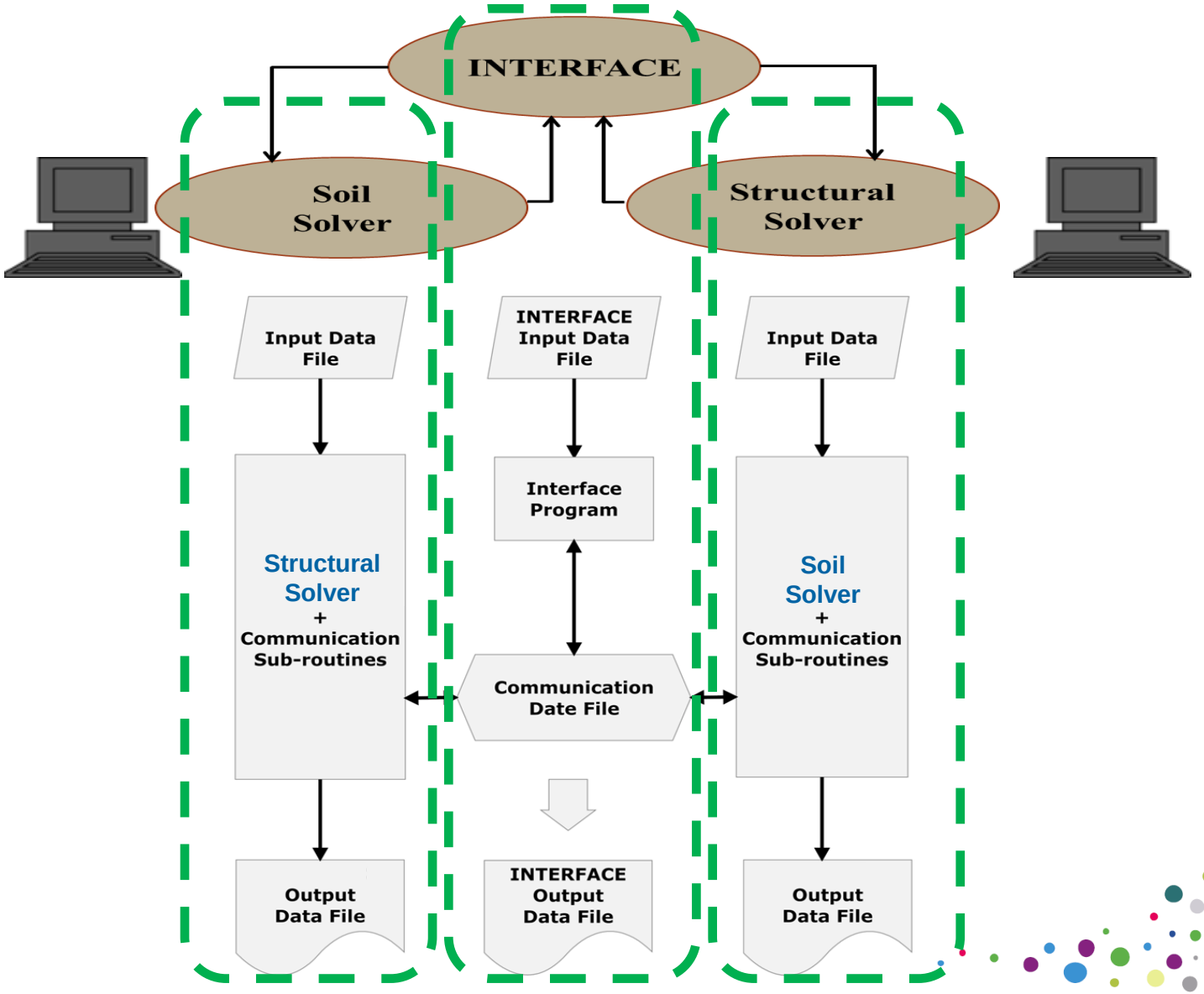


Detailed Response SSI Modelling

❖ Coupled Soil-Structure Interaction – Partitioned Approach:

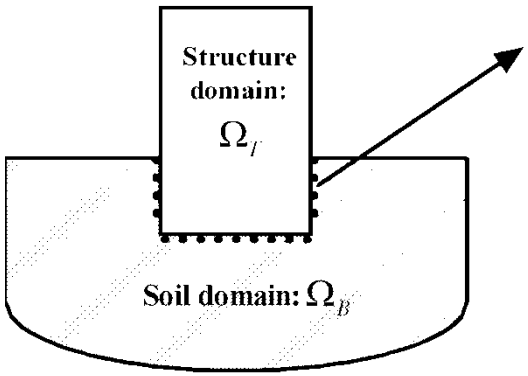
- Advanced Simulation Environment for SSI.
- computational methods for coupled modelling of SSI.
- SSI coupled modelling Applications

Simulation Environment



Soil-Structure Interaction

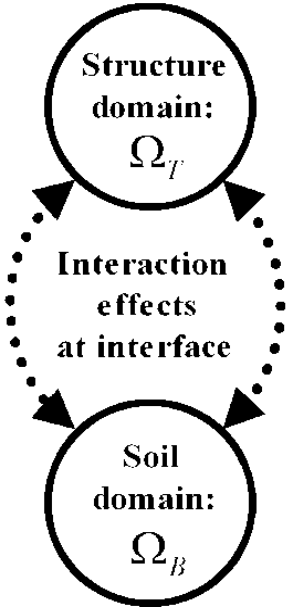
Coupling Formulation – Partitioning Strategy



Interface

$\Gamma_i = \Omega_T \cap \Omega_B$

$$\begin{bmatrix} K_{11}^T & K_{12}^T & 0 \\ K_{21}^T & K_{22}^T + K_{22}^B & K_{21}^B \\ 0 & K_{12}^B & K_{11}^B \end{bmatrix} \begin{Bmatrix} U_T^T \\ U^i \\ U_B^B \end{Bmatrix} = \begin{Bmatrix} F_T^T \\ F_{ext}^i \\ F_B^B \end{Bmatrix}$$



Structure sub-domain:

$$\begin{bmatrix} K_{11}^T & K_{12}^T \\ K_{21}^T & K_{22}^T \end{bmatrix} \begin{Bmatrix} U_T^T \\ U_T^i \end{Bmatrix} = \begin{Bmatrix} F_T^T \\ F_T^i \end{Bmatrix}$$

Soil sub-domain:

$$\begin{bmatrix} K_{11}^B & K_{12}^B \\ K_{21}^B & K_{22}^B \end{bmatrix} \begin{Bmatrix} U_B^B \\ U_B^i \end{Bmatrix} = \begin{Bmatrix} F_B^B \\ F_B^i \end{Bmatrix}$$

Soil-Structure Interaction

Coupling Formulation – Algorithm

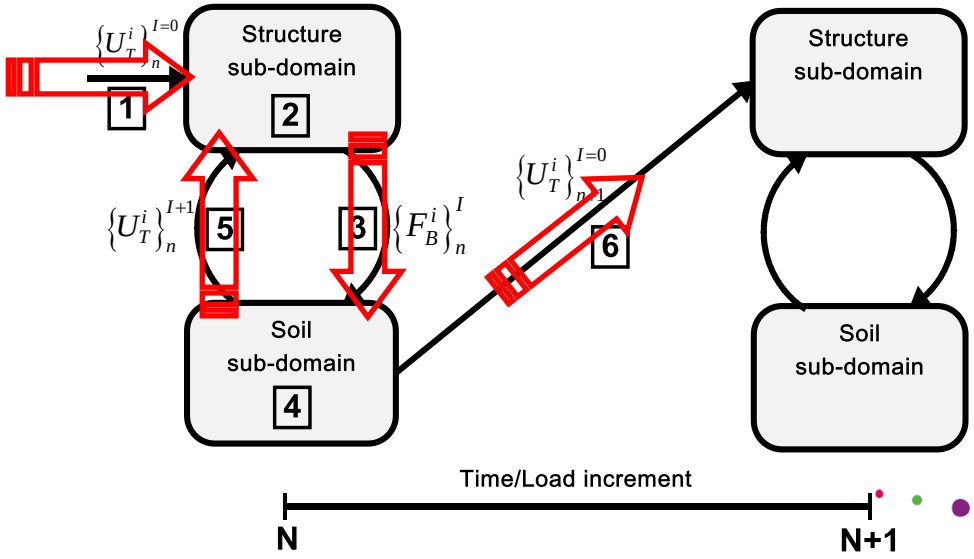
Sub-domain Ω_T (Structure):

$$\left(\frac{1}{\beta\Delta t^2}[M]_T+\frac{\gamma}{\beta\Delta t}[C]_T+[K]_T\right)\begin{Bmatrix}U_T^T\\U_T^i\end{Bmatrix}_{n+1}=\begin{Bmatrix}F_T^T\\F_T^i\end{Bmatrix}+\varphi\left(\begin{Bmatrix}\ddot{U}_T^T\\\ddot{U}_T^i\end{Bmatrix}_n+\begin{Bmatrix}\dot{U}_T^T\\\dot{U}_T^i\end{Bmatrix}_n+\begin{Bmatrix}U_T^T\\U_T^i\end{Bmatrix}_n\right)$$

Sub-domain Ω_B (Soil):

$$\left(\frac{1}{\beta\Delta t^2}[M]_B+\frac{\gamma}{\beta\Delta t}[C]_B+[K]_B\right)\begin{Bmatrix}U_B^B\\U_B^i\end{Bmatrix}_{n+1}=\begin{Bmatrix}F_B^B\\F_B^i\end{Bmatrix}_{n+1}+\phi\left(\begin{Bmatrix}\ddot{U}_B^B\\\ddot{U}_B^i\end{Bmatrix}_n+\begin{Bmatrix}\dot{U}_B^B\\\dot{U}_B^i\end{Bmatrix}_n+\begin{Bmatrix}U_B^B\\U_B^i\end{Bmatrix}_n\right)$$

Convergence Check



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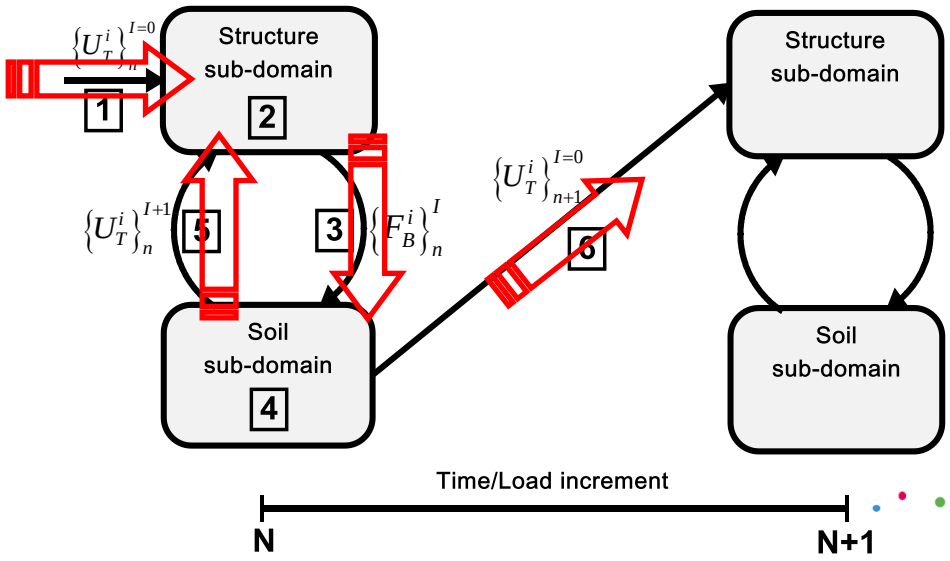
Soil-Structure Interaction

Coupling Formulation – Algorithm

?

How to achieve convergence through successive update of iterative boundary conditions at the interface



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Coupling Algorithms

Iterative Coupling Algorithms Formulation

$$\left\{U_{\text{struc}}^{\text{interface}}\right\}_n^{I+1} = [\alpha]\left\{U_{\text{soil}}^{\text{interface}}\right\}_n^{I} + \left([\mathbf{I}]-[\alpha]\right)\left\{U_{\text{structure}}^{\text{interface}}\right\}_n^{I}$$

Compatibility Error ***Error Reduction***

$$\left\{U_{\text{soil}}^{\text{interface}}\right\}^{K} - \left\{U_{\text{structure}}^{\text{interface}}\right\}^{K} = \left([\mathbf{I}]-[\alpha]\left([\lambda]+[\mathbf{I}]\right)\right)^{K}\left(\left\{U_{\text{soil}}^{\text{interface}}\right\}^{0} - \left\{U_{\text{structure}}^{\text{interface}}\right\}^{0}\right)$$

$$\left([\mathbf{I}]-[\alpha]\left([\mathbf{I}]+[\lambda]\right)\right)^{K} \rightarrow \mathbf{0}$$

1) Interface Relaxation

2) Reduced Order

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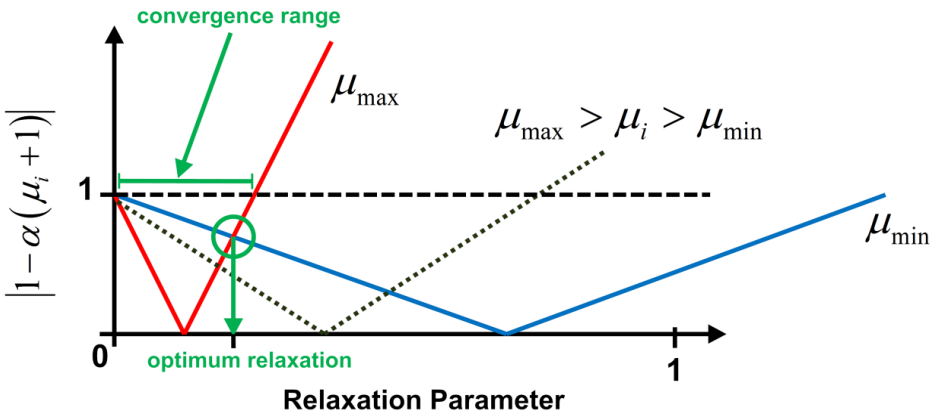
Iterative Coupling Algorithms Formulation

1-Interface Relaxation Approach



$$0 < \alpha < \frac{2}{1+\mu_{max}} < 2$$

$$0 < \alpha_{opt} = \frac{1}{1+\mu_{av}} = \frac{1}{1+\frac{\mu_{max}+\mu_{min}}{2}} \leq 1$$

2- Reduced Order Method

$$[\mathbf{U}_T] = \left[\{U_T^i\}^1 - \{U_T^i\}^I \quad \dots \quad \{U_T^i\}^j - \{U_T^i\}^I \quad \dots \quad \{U_T^i\}^{I-1} - \{U_T^i\}^I \right]_{M \times (I-1)}$$

$$[\mathbf{F}_T] = \left[\{F_T^i\}^1 - \{F_T^i\}^I \quad \dots \quad \{F_T^i\}^j - \{F_T^i\}^I \quad \dots \quad \{F_T^i\}^{I-1} - \{F_T^i\}^I \right]_{M \times (I-1)}$$

$$\{\Delta F_T^i\}_n^I \approx [\mathbf{F_T}] \left([\mathbf{U_T}]^T . [\mathbf{U_T}] \right)^{-1} [\mathbf{U_T}]^T \{\Delta U_T^i\}_n^I$$

$$\{\Delta F_T^i\}_n^I \approx [K_T^C] \{\Delta U_T^i\}_n^I$$

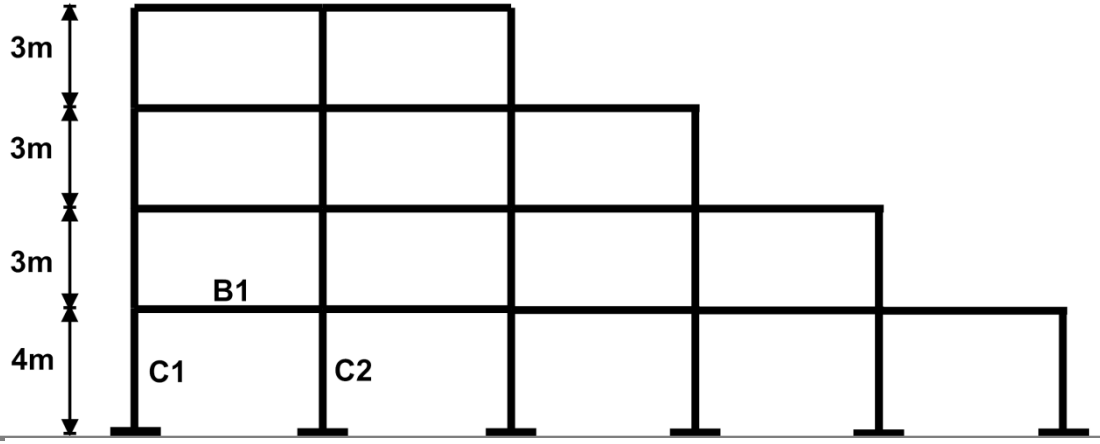
Coupling Algorithms

Iterative Coupling Algorithms Comparison

No	Method	Implementation	Convergence Rate	Nonlinear Problems	Modular Use of Softwares
1	Condensed Tangent Stiffness	* Difficult	******* Excellent	✓	-
2	Mixed Reduced Order	**** Fair	****** Excellent	✓	✓
3	Reduced Order	**** Fair	*** Good	✓	✓
4	Secant	*** Fair	***** Good(linear)	-	✓
5	Adaptive Relaxation	******** Easy	** Poor	-	✓
6	Constant Relaxation	******** Easy	* Poor	-	✓

Detailed Response Modelling

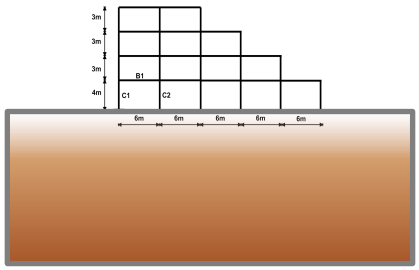
❖ Settlement Analysis of Multi-storey Five-bay Steel Frame

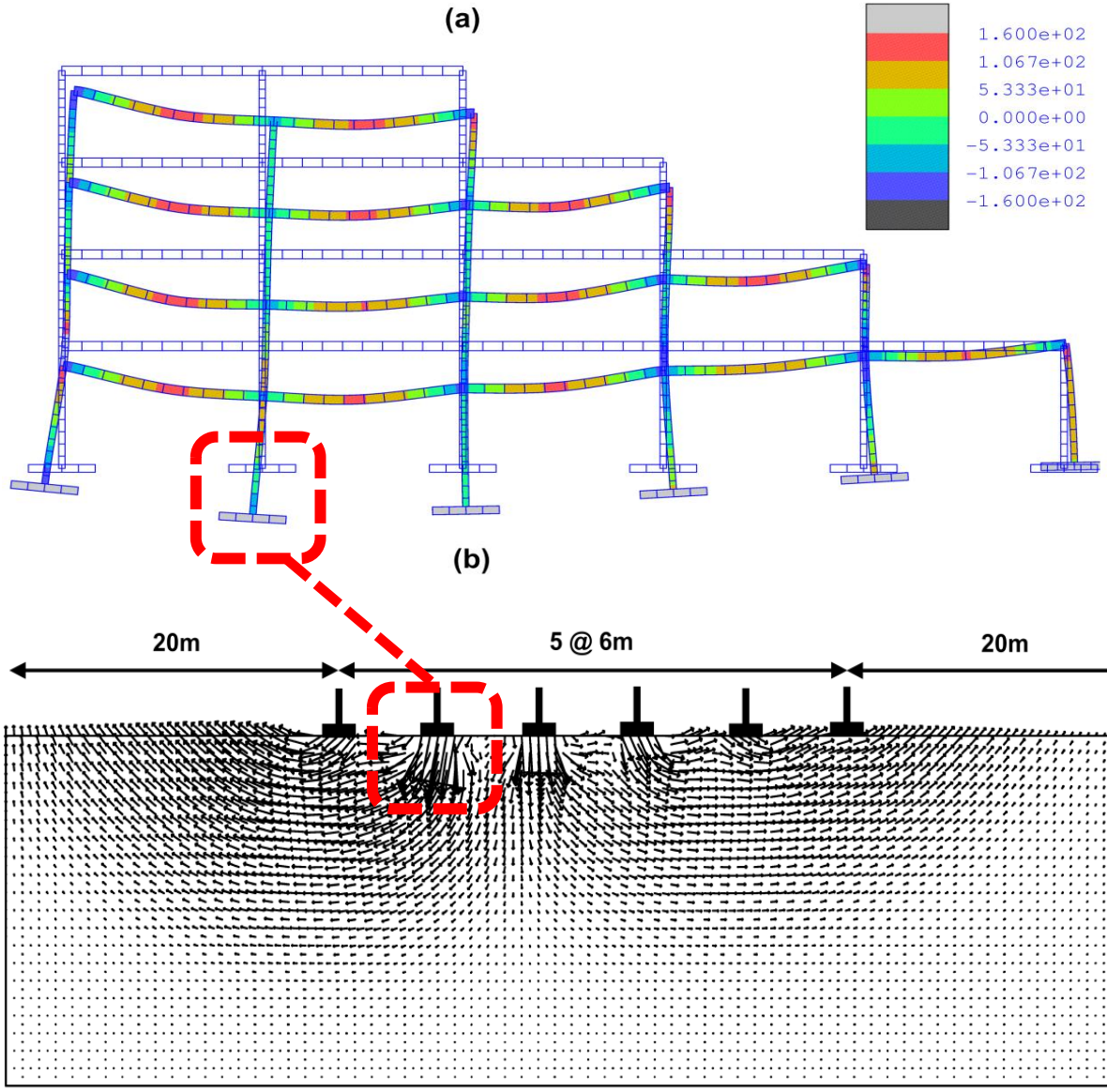


Structure Sub-domain	Material Properties
Columns	UC 203×203×46
Beams	UB 305×102×25
All beams and columns (steel)	Steel Grade = S355 Elastic Modules = 210 GPa Strength = 355 MPa Bilinear elasto-plastic with strain Hardening Factor = 1%
Foundation Beam (concrete)	Elastic Modulus = 30 GPa Linear material Size: 2m×0.5m
Soil Sub-domain	**Material Properties**
Soil	Angle of Shear resistance (Φ') = 22° Dilation angle (ν') = 11° Effective out of plane depth = 1m Cohesion = 20 kPa Young's modulus varies linearly with depth from 10000 kPa at the ground surface (dE/dZ=5000 kPa/m) Elasto-plastic Mohr-Coulomb constitutive model

Detailed Response Modelling

❖ Settlement Analysis of Multi-storey Five-bay Steel Frame





Detailed Response Modelling

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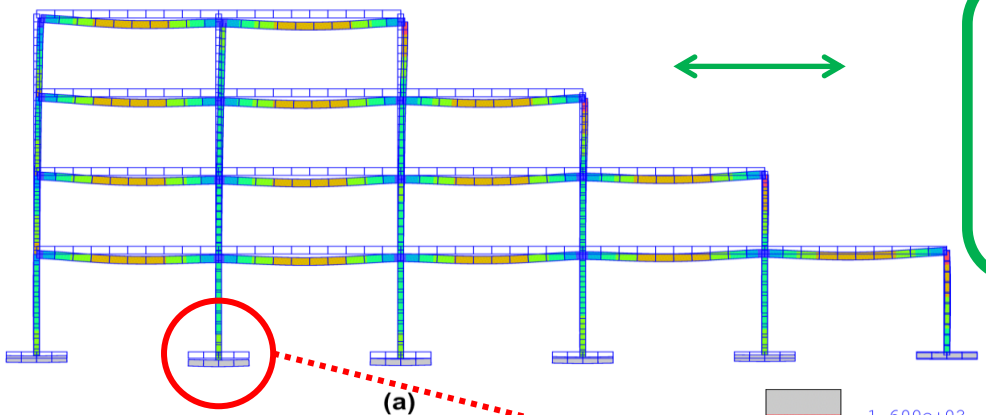
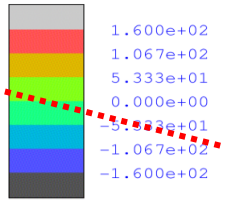
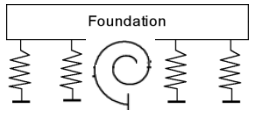
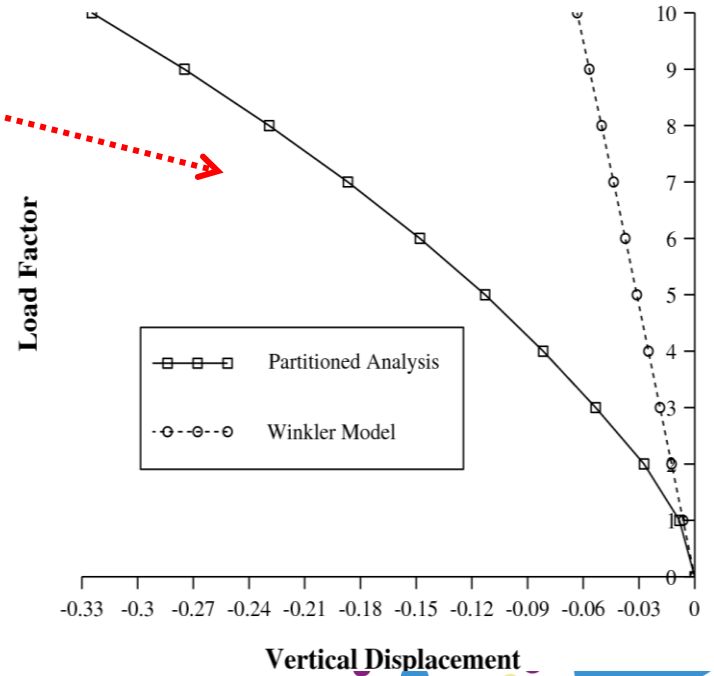




Field Elimination – Winkler Foundation



Detailed – Partitioned Approach



Detailed Response Modelling

❖ Building Response to an Adjacent Un-braced Excavation

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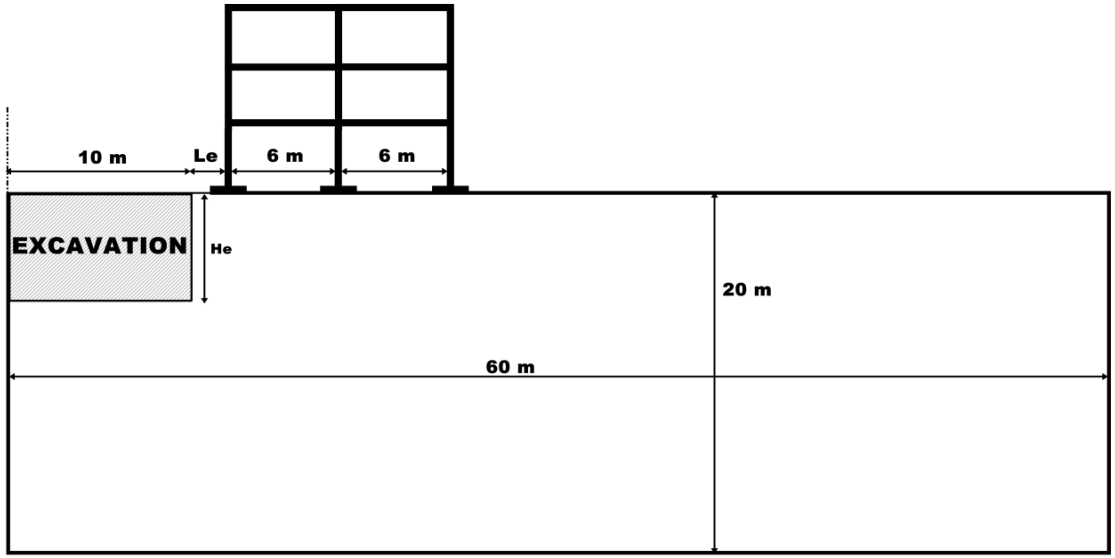
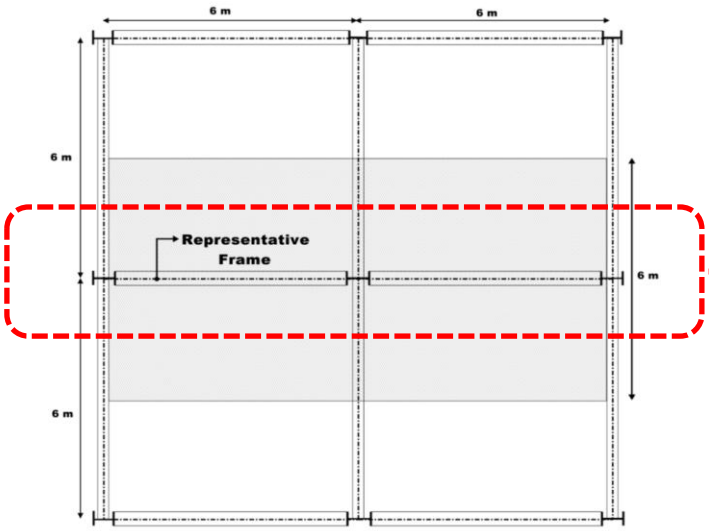
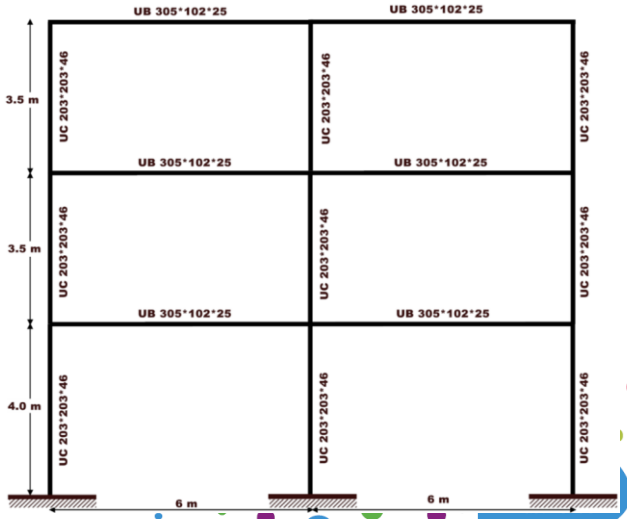
Detailed Response Modelling

❖ Building Response to an Adjacent Un-braced Excavation

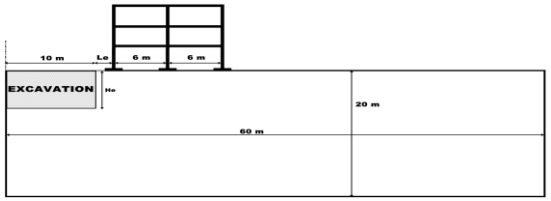


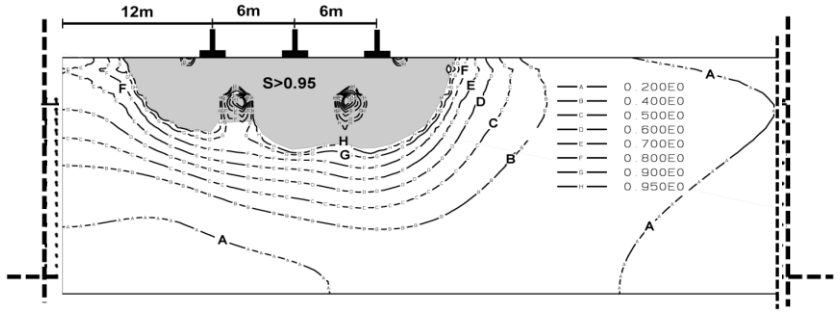


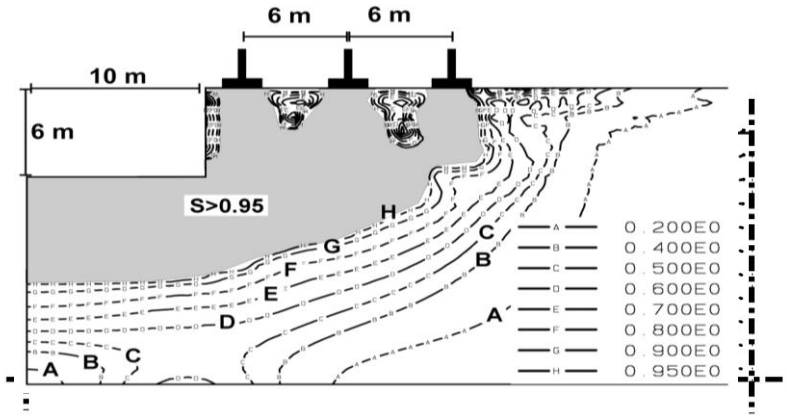


Detailed Response Modelling

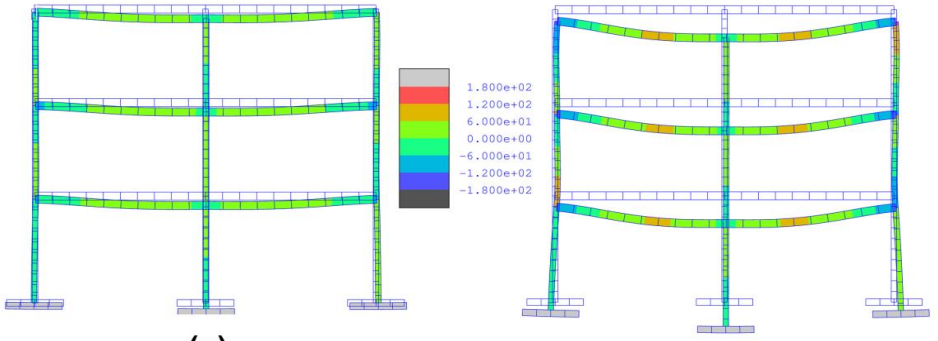
❖ Building Response to an Adjacent Un-braced Excavation



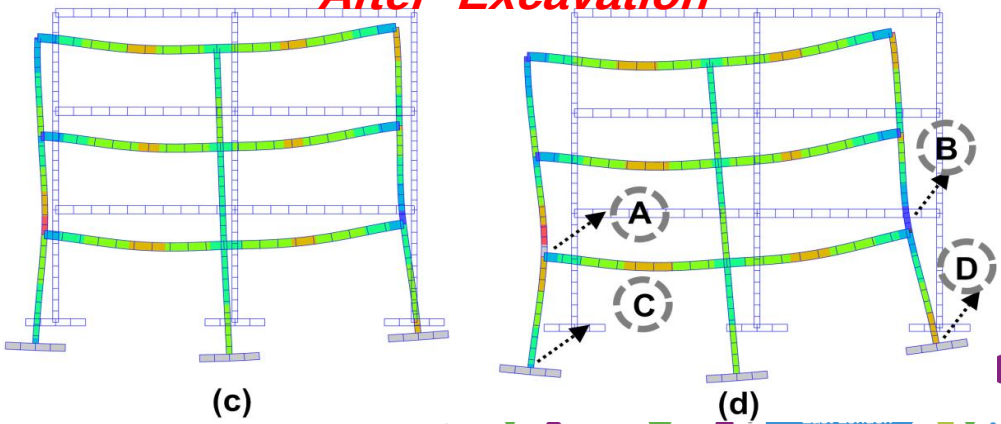




Before Excavation



After Excavation

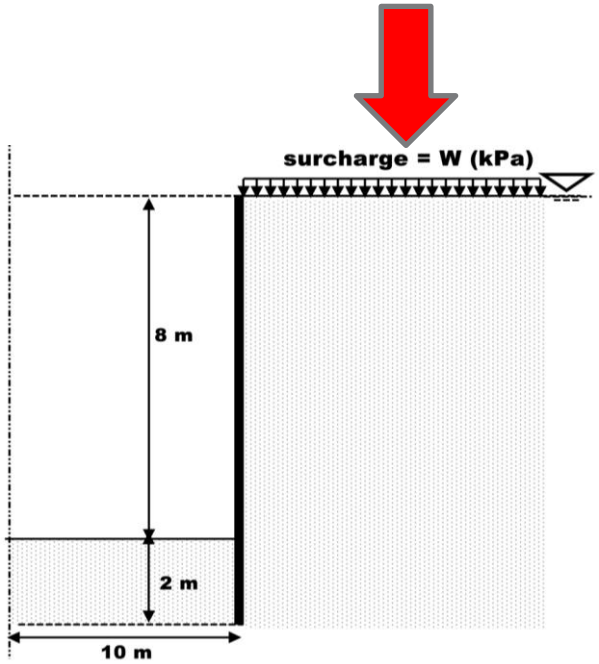


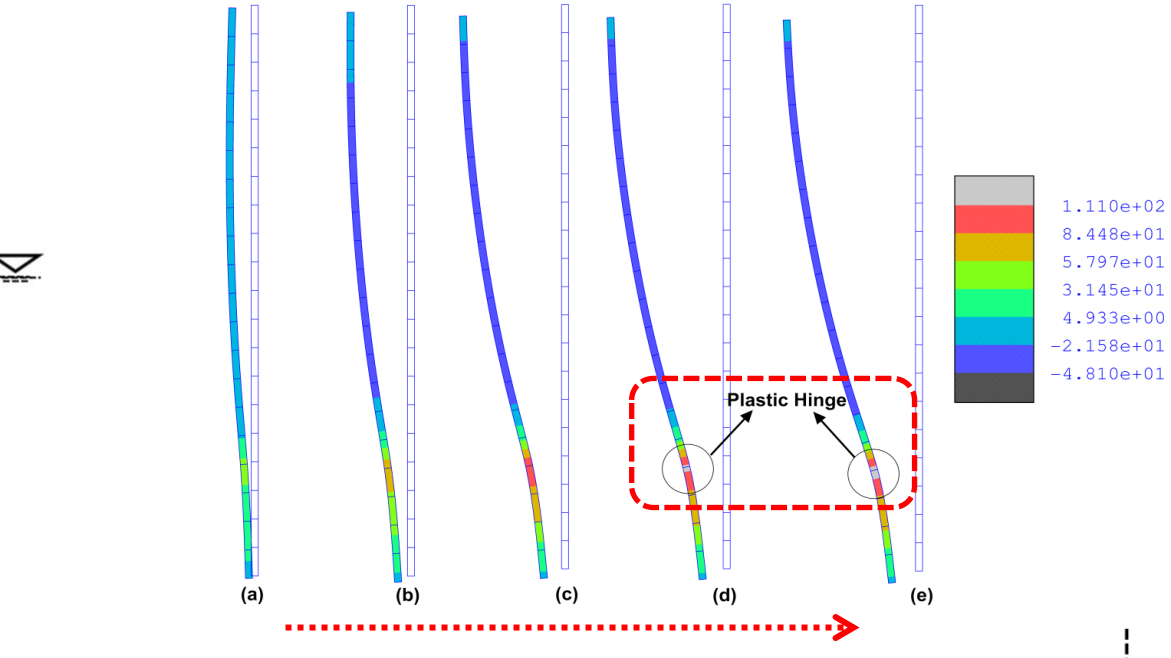
Detailed Response Modelling

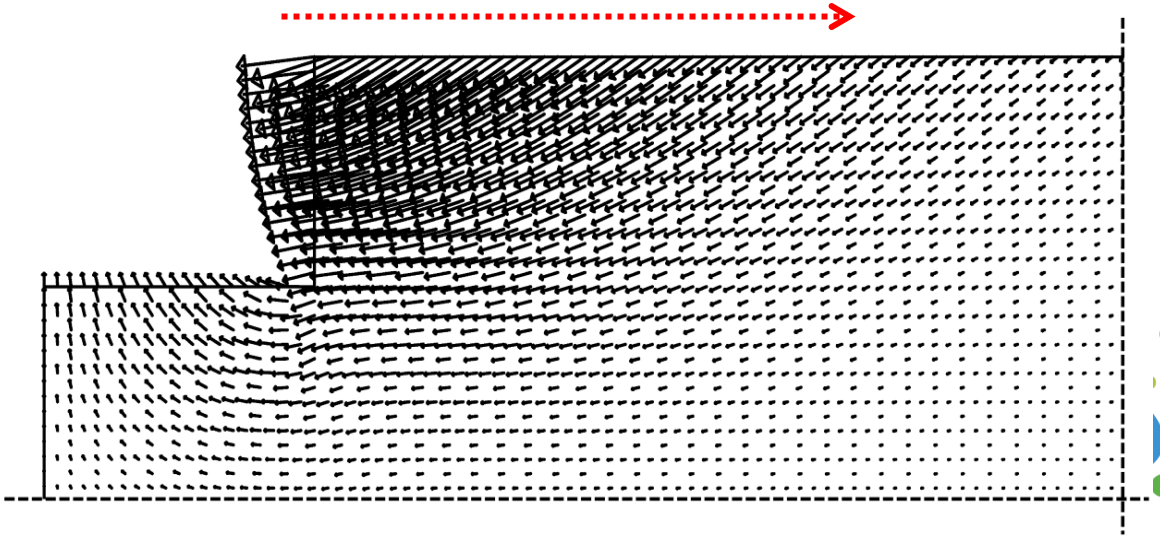
❖ Nonlinear Steel Sheet Pile Retaining wall modelling

Detailed Response Modelling

❖ Nonlinear Steel Sheet Pile Retaining wall modelling







Concluding Remarks

❖ Engineering Research

- ❑ Explore some extremely complex phenomena in the field of Engineering
- ❑ Study those in a rigorous way

& just as importantly

- ❑ look for ways to communicate the results
  - ➢ Aid on understanding the subject area
  - ➢ Ensure the outcomes would be of assistance in the practice

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Thank You!