HYDRAULIC ROUGHNESS - LINKS BETWEEN MANNING'S COEFFICIENT, NIKURADSE'S EQUIVALENT SAND ROUGHNESS AND BED GRAIN SIZE

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Abstract: This paper presents and reviews the connection between Manning's n and Nikuradse's equivalent sand roughness k_s, which is well established for pipe flow in the rough turbulent region. The link with bed grain size is less clear, and a survey is made covering pipelines and channels, river and coastal engineering. It is concluded that whilst the equivalent n and k_s values are useful alternatives for sewer and culvert design, the link between roughness parameters and bed grain size for river and coastal purposes should be treated with more caution, particularly because the hydraulic resistance is likely to include not only a skin friction element which depends on the grain size, but also a form drag component.

Notation

d	sediment grain diameter (subscripts indicate percentage finer)
f	wave friction factor
g	acceleration due to gravity
k_s	surface roughness height, Nikuradse's equivalent sand roughness
k	alternative for k_s
n	Manning's roughness coefficient
r	hydraulic roughness in coastal waters
D	diameter of pipe
K	alternative Manning coefficient = 1/n
R	hydraulic radius (cross section area of flow divided by wetted perimeter)
S	hydraulic gradient, head loss per unit length
T	wave period
U	maximum wave orbital velocity
V	average velocity of flow
φ( )	function of
λ	Darcy friction factor ($= 2gDS/V^2$)
τ	bed shear stress
ω	wave frequency [radians] ($= 2\pi/T$)
[ ]	units

1. Introduction

Various friction formulae are used in different contexts for hydraulic calculations, and it is instructive to compare the different friction factors used, for ease of converting from one formula to another. It is also useful to consider the link to the bed material, for the purpose of predicting the roughness of a channel. This paper focuses on the Manning formula and the Colebrook White equation which are both widely used in practice. A practical user-oriented review is given, rather than a complete historical account of the development of the relevant theory.

2. Hydraulic theory

Manning's formula is widely used in river engineering and drainage applications, and is particularly applicable to the rough turbulent region of flow. Traditionally this uses a roughness coefficient n in the following formula in SI units:

$$V = \frac{1}{n} R^{2/3} S^{1/2} \qquad (1)$$

The term (1/n) needs units of $m^{1/3}/s$ to balance dimensionally. In US publications using the Imperial length unit of feet, this term is replaced by (1.486/n), and the numerical values of n are the same as used when working in the SI or metric system.
Application to a full pipeline of diameter D is easily achieved using the following relationship for the hydraulic radius of a circle:

$$R = \frac{A}{P} = \frac{D}{4} \qquad (2)$$

The second widely used formula that will be considered is Colebrook White, which relates the Darcy friction factor λ to the relative roughness and Reynolds number

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{k_s}{3.7D} + \frac{2.51}{Re\sqrt{\lambda}}\right) \quad (3)$$

In the rough turbulent region, where variations due to changes in Reynolds number are negligible, equation (3) may be simplified to the rough turbulent version:

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{k_s}{3.7D}\right) \qquad (4)$$

Note that the logarithm in equations (3) and (4) is to base 10, and the negative sign is necessary on the right hand side of the equation to return a positive value.
Combining equations (1), (2) and (4) with the definition of λ from the notation, one may obtain the expression:

$$\frac{k_s^{1/6}}{n} = \frac{2\log(3.7D/k_s)\, 4^{2/3}\sqrt{2g}}{(D/k_s)^{1/6}} \qquad (5)$$

Although this appears involved, in fact over a typical range of relative roughness values

$$0.001 \le \frac{k_s}{D} \le 0.01 \qquad (6)$$

it may be shown (for example in Webber (1971) p.101, also in UEL module CEM001 notes) that the right hand side of equation (5) is reasonably constant, and so there is a useful approximate relationship between Manning's n and the surface roughness k_s as follows, with the latter expressed in metres:

$$n \approx \frac{(k_s[m])^{1/6}}{26} \qquad (7)$$

$$n \approx 0.038(k_s[m])^{1/6} \qquad (8)$$

Alternatively, with the surface roughness expressed in millimetres:

$$n \approx 0.012(k_s[mm])^{1/6} \qquad (9)$$

Some adjustment to the above would result from using the wide channel version of Colebrook White (which factors the terms within the log bracket in equation (3) by 1.203, and replaces D by 4R, so that $k_s/3.7D$ is replaced by $k_s/12.3R$ etc.), and that would make the denominator in equation (7)

approximately 25, the factor in equation (8) 0.040 and the factor in equation (9) nearer to 0.013. Ackers (1958) referred to this point, and considered this correction unimportant for practical purposes since the roughness for design purposes can seldom be estimated to an accuracy of better than 20 percent.
The above equations (7) to (9) therefore give a reasonable relationship between Manning's n and the surface roughness height k_s, but the question remains how k_s is related to the grain size of the bed material. This is reviewed in the following sections, in the context of pipelines and channels, rivers and coastal engineering applications.

3. Pipelines and channels

The measure k_s derives from the work of Nikuradse (published in German in the 1930s), who glued uniformly sized sand grains to the internal surface of pipes, and indeed the term is sometimes referred to as the equivalent sand roughness. One might suppose therefore that this could be set equal to the grain size, but there are the added complications of spacing, pattern and variation of size of the grains, as well as the possible mobility of the sediment. It is also important to remember that Nikuradse's investigation dealt only with surface or skin friction drag caused by the bed grains, and not the form drag arising from bedforms such as ripples and dunes.
The Wallingford Tables and Charts (HRW 1990) included the comment that "The k_s values bear some relation to the physical dimension of the roughness projections, and therefore a visual examination of a particular surface will give a guide to its roughness. Strickler's investigation of natural channels indicated that the k_s value corresponded to the size which was exceeded by 10 percent of the bed material..."

Butler, May and Ackers (1996) in their work on sediment transport in sewers adopted the following relationship for flow in circular pipes with a deposited bed of sediment:

$$k_s = 1.23 d_{50} \qquad (10)$$

In answer to a discussion item on that paper, the authors explained that the reason for this relationship was primarily a wish to achieve commonality with the earlier sediment transport work of Ackers and White. They also claim this result agreed satisfactorily with their experimental data.
Naqvi (2003) draws on earlier work by Williamson to produce a version of equation (7) with 26.3 on the denominator, but when the approximation involved is considered, three significant figures may be thought too precise. This links Manning's n with the surface roughness height of drainage channels, but without any link to the size of sediment that may possibly be present. This may well be because the channels are assumed to be sediment free or self cleansing.
Both the Manning formula and Colebrook White are recommended in the latest sewerage standard BS EN 752 (2008), and it is noted that as in a number of European publications, the term (1/n) in the Manning equation is replaced by K [$m^{1/3}/s$], not to be confused with the surface roughness height k_s , which is given simply as k. It is noted that Colebrook-White equation in the British version is named Colebrook in the French version and Prandtl-Colebrook in the German version. The Manning equation in the British version is named Manning-Strickler in the French and German versions. Recommended values are given for the Manning coefficient for use in drainage channels (K = 55 to 90 $m^{1/3}/s$), and for the pipeline roughness (k = 0.6 or 1.5 mm) for use in sewer pipes. Similar to equation (5),

a conversion expression is included, but beware that contains a typographical error. There is no recommended link to sediment size.

4. River engineering

Some texts such as Nalluri and Featherstone (2001) covering flow in loose-boundary channels, replace k_s by the typical sediment grain diameter d in equation (7), and refer to this as Strickler's equation. However in other texts, various versions of Strickler's equation appear, with differences in the numerical value and the representative grain size. Chow (1959) gave the relationship

$$n = \varphi(R/k)k^{1/6} \tag{11}$$

and quoted Strickler (1923) from observations made in Switzerland arriving at an average value for the function φ(R/k) = 0.0342, with the roughness height being the median sieve size of the material, expressed in feet. When the units are converted for the size expressed in metres, the constant becomes 0.0417 or 1/24.

Webber (1971) included the derivation of equation (7) above with 26 as denominator, but then quoted Strickler's equation as

$$\frac{d_{50}^{1/6}}{n} = 24.1 \tag{12}$$

with d_{50} in metres, which is close to the value deduced from Chow.

Vischer and Hager (1998) quoted Strickler (1923) as an empirical relationship for bottom slopes $0.004 < S_0 < 0.025$ and hydraulic radii 0.1 m < R < 10 m, as follows

$$n = \frac{d_{50}^{1/6}}{21.1} \tag{13}$$

again with d_{50} in metres, which gives a higher value of n than in previous equations.

Vischer and Hager also included a formula by Meyer-Peter and Müller (1948) – with what must be a typographical error of a minus sign in the power of d_{90} which has been removed

$$n = \frac{d_{90}^{1/6}}{8.2\sqrt{g}} \tag{14}$$

In metric units with g = 9.81 m/s^2 this equation (14) yields a similar result to equation (7) above but in this case using d_{90} rather than d_{50} in place of k_s.

Smart (1999) quoted results from Canada that found the best prediction was obtained by taking d in Strickler's equation as 6.75 times the median diameter of the local bed material.

Strickler's equation is quoted by Smart as

$$n = \frac{d^{1/6}}{6.7\sqrt{g}} \tag{15}$$

where d is a representative size for the channel bed material. Taking g = 9.81 m/s^2, this gives a denominator of 21, which is similar to equation (13), but if 6.75 d_{50} is substituted for d, then the equation becomes considerably different.

Smart (1999) also quoted from Van Rijn that "grain size based equations are not precise, and an order of magnitude variation in the relationship between hydraulic roughness and a representative bed particle size is often reported for alluvial bed channels". Smart concluded that there may be a problem trying to base resistance formulae for alluvial channels on theory and experiments for fixed impermeable boundaries.

Sturm (2010) includes a review which also illustrates the range of values quoted, and makes the additional important point particularly for US publications when considering data where k_s is measured in feet rather than metres. This introduces a factor of $(0.3048)^{1/6} = 0.82$, and this may not be immediately apparent when considering versions of the above equations.

5. Coastal engineering

In coastal engineering there is no application of Manning's equation as such, but friction factors and bed grain size are relevant to bed stability and sediment transport considerations. A typical definition for bed shear stress τ is

$$\tau = \frac{1}{2}\rho f U^2 \qquad (16)$$

where U is maximum wave orbital velocity and f is friction factor. It may be shown from this definition that 4f is equivalent to λ as used above.
The wave friction factor f may be expressed by equations such as that given by Swart (1974):

$$f = \exp\left(5.213(r/A)^{0.194} - 5.977\right) \qquad (17)$$

where r is hydraulic roughness and A is the orbital amplitude of fluid just above the boundary layer. (r/A) is thus a type of relative bed roughness term, which Swart proposed in fully developed turbulent flow to be

$$\frac{r}{A} = \frac{5\pi d}{TU} \qquad (18)$$

with T as the wave period, equal to $2\pi/\omega$ where ω is frequency. Since $U=A\omega$, it may be seen that equation (18) is equivalent to taking r = 2.5 d. Other publications also give r as a function of d, to represent the skin friction due to waves in rough turbulent conditions. However, as Nielsen (1992) indicated, for oscillatory flows over a sandy bed, the hydraulic roughness is generally one or two orders of magnitude bigger than the equivalent sand size, due to bed forms with significant ripple height.
There is extensive literature in this area, which space does not allow us to cover here, and which would require a separate paper to do justice to the wealth of empirical results. Suffice it to say that a number of empirical relationships for hydraulic roughness in coastal waters are expressed in terms of both sand grain diameter and ripple dimensions.

6. Conclusions

- The link between Manning's n and Nikuradse's k_s is well established for flows in the rough turbulent region, as given for example in equation (7).
- It is important to specify the units involved for k_s.
- A typical sediment grain diameter or multiple thereof may be used in place of k_s, but a wide range of values are quoted in different publications, showing that this is not a precise relationship.
- Considering the range of values quoted, it is suggested that an appropriate general relationship approximated to one significant figure, to link Manning's n with typical sediment diameter would be

$$n \approx 0.04(d[m])^{1/6} \qquad (19)$$

- Results to more significant figures may apply to particular data sets, but the implied precision should be treated with caution.

- Relationships such as equation (19) should also be treated with caution where mobile beds are involved, since the hydraulic resistance is likely to involve not just skin friction but also form drag caused by bedforms such as ripples and dunes. This is particularly relevant to river and coastal applications.

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