Empowering Firms with AI-Generated Content: Strategic Approaches to R&D and Advertising in the Era of Generative AI

Abstract — Commercial utilization of Generative Artificial Intelligence (GAI) is expanding rapidly. However, few studies have investigated the transformative impact of GAI on business operations, with a specific focus on Research and Development (R&D) and advertising facilitated by AI-generated content (AIGC) empowerment; particularly, there is a lack of analysis on the impact of AIGC iteration on firms' decision-making. The study uses the optimal control and learning-by-doing method to examine two types of AIGC strategies, the single AIGC (enhancing R&D exclusively) and the dual AIGC (simultaneously boosting R&D and advertising), to delve into their dynamic iteration effects on firms' performance. Our findings reveal that, under the single AIGC strategy, focusing on investments in the AI training R&D sector alone can enhance the GAI smartness level more effectively than adopting the dual AIGC strategy. Conversely, under the dual AIGC strategy, firms initially tend to select GAI empowerment toward the downstream advertising sector more than the upstream R&D sector. Both strategies demonstrate enhanced profits and increased demand with higher rates of AIGC self-learning. Notably, considering the limited budget, firms prioritize allocating AI training resources to the R&D sector under the dual AIGC strategy, guaranteeing their long-term success.

Index Terms—Generative artificial intelligence, artificial intelligence-generated content, artificial intelligence training, self-learning, research and development, advertising.

Managerial Relevance Statement—Our findings bear the following practical implications. First, our findings reveal that under the dual AIGC strategy, the self-learning rate in the upstream R&D sector exhibits a notable "spillover effect." Surprisingly, this effect is not observed in the downstream advertising sector. This highlights the critical role of Generative AI (GAI) in empowering the upstream R&D sector, which is more influential than its application in the advertising sector. Therefore, engineering managers should prioritize implementing GAI in the R&D sector rather than the advertising sector, as R&D-driven GAI adoption positively impacts other sectors, including advertising.

Second, the results indicate that under the single AIGC strategy, the rate of change in AI training efforts in the R&D sector decreases as its self-learning rate increases. This finding underscores the dual influences on R&D-based AIGC empowerment: an external factor (training GAI) and an internal factor (GAI's self-learning capability). These factors interact dynamically during the firm's GAI evolution, suggesting that firms should intensify external GAI training efforts in the early stages before GAI attains greater intelligence and autonomy in R&D tasks.

Third, our research demonstrates that as R&D self-learning rates improve, both AI smartness and the firm's goodwill increase. This finding highlights that R&D-empowered

GAI indirectly and positively influences the firm's goodwill, which is often a primary focus of advertising-empowered GAI. It suggests that the dual AIGC strategy achieves a synergistic effect, where the combined benefits are greater than the sum of individual contributions ("1+1 > 2"). Consequently, adopting a dual AIGC strategy offers superior outcomes compared to relying on a single AIGC approach.

Fourth, we also find that the contribution rate of investments to AI smartness is higher under the single AIGC strategy than under the dual AIGC strategy. This implies that focusing solely on AI training in the R&D sector can more effectively enhance AI intelligence levels. However, under the dual AIGC strategy, the advertising sector's investment contributions to goodwill surpass the R&D sector's contributions to AI smartness. This finding suggests that engineering managers must carefully allocate training resources between the two sectors when implementing a dual AIGC strategy to maximize overall benefits.

Ⅰ. INTRODUCTION

The expansion of ChatGPT, a Generative Artificial Intelligence (GAI), has ignited a significant societal debate [1]. In particular, this technology holds the remarkable potential to profoundly reshape the business landscape, thereby impacting firms' operations by accelerating the design process, improving supply chains, and offering solutions to advertising and marketing. To harness GAI's full potential, firms worldwide have enthusiastically adopted and integrated GAI into their business operations [2]. Major players like *Google*'s PLAM2, *Microsoft*'s BingChat, *Amazon*'s Titan, *Huawei*'s PanGu, and *JD*'s ChatJD have embraced the GAI powered by Large Language Models (LLMs), with an aim to cut related costs, boost productivity, and sharpen their competitiveness. These firms come across diversified industries, including automobile, electronics, household appliances, clothing, and pharmaceuticals, characterized by high customization, short product life cycles, and fast technology iteration. All of these attributes require firms to invest in training GAIs to reach the smartness level continuously. In this regard, GAIs acquire the smartness ability of self-learning and decision-making like human beings, which enables these firms to autonomously and quickly generate pragmatic ideations and solutions, thus enhancing the efficiency of firms' operations [3].

In recent years, two typical categories of GAIs have been widely applied in business operations. One is the R&D-empowered GAI, and the other is the advertising/ marketing-empowered GAI. For the former, GAI is used to create, optimize, and auto-complete R&D solutions tailored to customers' needs; we refer to this AI-generated content (AIGC) as R&D-based AIGC. With the R&D-based AIGC, engineers and project managers can work through the design process much faster and more efficiently than ever before to assess ideas based on the constraints of the project. For example, *Huawei*, as a digital giant, has explored a type of GAI called PanGu, thus revolutionizing the bio-pharmaceutical sector by R&D-empowered GAI identifying the most promising solutions for new drug R&D amidst vast alternatives, thereby significantly enhancing the success rates, reducing the R&D costs and shortening the innovation cycle [4].

For the latter, GAI is used to generate text and images needed for advertising and marketing or finding new ways to interact with customers; we refer to them as advertising/marketing-based AIGC. With advertising/marketing-based AIGC, this powerful recommendation engine helps firms discover new customers who match their products. For example, *JD* leverages its own GAI, ChatJD, to devise highly effective target marketing strategies for its platform firms, effectively positioning their products to reach the most promising consumers, thus strengthening the relationship with customers and benefiting the firms' sales and brand reputation [5].

Regardless of whether it is R&D-based AIGC or advertising/marketing-based AIGC, both have an iteration effect. As we know, AIGC renewals need firms' continuous training of GAI to facilitate the accumulation of relevant knowledge. The knowledge accumulation, in return, improves the GAI self-learning and decision-making abilities; in this way, GAI becomes smarter, and the related AIGC is continuously updated over time. The whole cyclic procedure is a kind of AIGC iteration process, where the corresponding AIGC evolves and iterates, enabling firms to more quickly and accurately respond to market changes in a cost-saving and efficient way; we call this phenomenon the AIGC iteration effect.

Although R&D-based AIGC and advertising/marketing-based AIGC are two primary ways to enhance firms' operation efficiency via such iteration effect [6] [7], firms are still perplexed about whether to adopt AIGC strategy in the current infant AI stage or wait until it matures before adoption, because the training GAI needs a huge amount of money. In addition, with the limited budget for training GAI, prioritizing the application of upstream R&D-based AIGC, downstream advertising/marketing-based AIGC, or both is also challenging for firms [8]. For example, *Shein*, a Singapore-headquartered global online fashion retailer, offers a full range of ever-evolving products centered around customers' needs. To meet the diverse needs of global customers more accurately and efficiently, *Shein* is determined to adopt AIGC via GAI but faces the challenge of selecting a portfolio investment on AIGC [53].

Thus, understanding the different roles of the R&D-based AIGC and the advertising/marketing-based AIGC strategy in firms' decision-making process can provide helpful guidance for firms' operations. However, the current literature on AI or AIGC-related studies mostly focuses on the implications of AI on innovation and user experience. For example, Sauvola *et al*. [9] and Alexander *et al*. [10] emphasize the importance of AI innovation, while Govindan [11] and Grashof and Kopka [12] explore the impacts of AI innovation. Li and Lee [13] and Khan and Mishra [14] investigate various factors influencing consumer-AI experiences from the AI marketing perspective. However, very few studies have considered the specific kinds of AIGC empowered by GAI and their AIGC iteration effect from a dynamic process perspective.

Inspired by the above discussion, this research aims to develop the GAI analytic framework, where the AIGC iteration effect and the capital constraint are considered under two strategies, i.e., the single AIGC strategy and the dual AIGC strategy. The single AIGC strategy refers to one that the firm merely adopts the R&D-based AIGC strategy, while the dual AIGC strategy is one that the firm simultaneously adopts the R&D-based AIGC and the advertising/marketing-based AIGC; in other words, the firm extends from the R&D-based AIGC to the advertising/marketing-based AIGC. Therefore, this study explores the following three questions:

RQ1: *How does the AIGC iteration affect a firm's R&D and advertising/marketing sectors under two AIGC strategies?*

RQ2: *What time and under what conditions does a firm select a specific AIGC strategy under the AIGC iteration effect*?

RQ3: *With limited budgetary resources, how does a firm allocate training in GAI capital under the dual AIGC strategy*?

To answer the above three questions, we build an optimal-control theoretical model, where the firm's GAI iteration process over time is formulated by learning-by-doing differential equations to characterize the evolution of AIGC through AI training and self-learning; meanwhile, we also employ the Hamiltonian function to compute the models under two AIGC strategies.

The main novelties of this research are threefold: (i) We offer pioneering insights into adopting two typical AIGC strategies under the AIGC iteration effect. While prior literature [9, 10, 13, 14, 23] has explored the impact of generic AI on firm performance after AIGC emergence, it has rarely delved into the specific AIGC strategies, let alone consider the AIGC iteration effect. (ii) Unlike previous research that focused AI applications on a single sector [13, 24, 25, 32, 36], we examine the GAI across two distinct sectors, the upstream R&D sector and the downstream marketing sector, and analyze the interaction between two sectors when adopting the dual AIGC strategy. (iii) This research first employs the Optimal Control Model and Learning-by-doing method to characterize the AIGC iteration effect dynamically and the relationship between GAI training cost, self-learning rate, and GAI smartness level, which rarely utilized in previous AI-related studies [9, 10, 12,13, 14].

The remainder of the paper unfolds as follows. Section Ⅱ provides a brief review of the related literature. The subsequent two sections introduce analytical models for two typical AIGC strategies and compare the outcomes. Section V presents a numerical experiment. Then, section VI extends the model to demonstrate theoretical contribution and managerial implications in section Ⅶ. Section Ⅷ draws conclusions. All proofs are detailed in the Appendix.

Ⅱ. LITERATURE REVIEW

GAI, as a specific AI, is different from Analytical AI and Discriminative AI. It generates and creates text, images, audio, and video information through data training in large language models (LLMs). AIGC is the application representative form of GAI, offering solutions or suggestions for firms [8]. Currently, a considerable body of research has explored different AIGC applications in various settings, including education [3], healthcare [16], and law [17], among others. The literature closely related to our work mainly spans three areas: AI innovation, AI marketing, and optimal control model in advertising. Thus, we review these three streams of work.

A. AI Innovation

The current literature on AI innovation in operational management focuses on two domains: its role (i.e., the practical lens) and related impact (i.e., the academic lens). For the first domain, Alexander *et al*. [10] claim that AI innovation can be applied to all sectors of the manufacturing process, from product design optimization to anomaly detection for quality control. Sauvola *et al*. [9] analyze the potential of AI innovation that has been proven to improve productivity, cost, and quality from creative dimensions to replacing repetitive and manual tasks. Yang and Wang *et al*. [18] propose an AI innovation-based framework for parallel manufacturing. Zheng et al. [19] find that AI innovation can fundamentally give rise to new modes of mass-customized production.

For the second domain of AI innovation, i.e., the related impact, Grashof and Kopka [12] empirically demonstrate the transformative effect of AI technology on radical innovation by studying large enterprises and SMEs, both reaping benefits from its applications. Similarly, Tekic and Füller [20] highlight how AI significantly reshapes firms' innovation processes, fostering more open, collaborative approaches and novel strategies for innovation protection. Broekhuizen *et al*. [21] and Freisinger *et al*. [22] delve into AI-based open crowdsourcing innovation. In particular, Broekhuizen *et al*. [21] systematically analyze AI's utilization for complex and unstructured tasks in open innovation scenarios, while Freisinger et al. [22] investigate human crowdsourcers' adoption behaviors in human-AI augmentation setups. Moreover, Zhou, Zhang, and Yu [23] analyze the iterative problem-solving process of AI-based innovation, empirically examining its impact.

The AIGC innovation-related work mainly focuses on AIGC copyrights and AIGC users. For example, Chesterman [24] is concerned with the ownership of the AIGC and explores the implications of related policy choices. Liu et al. [25] consider that AIGC products are still unprotected and vulnerable to tampering and plagiarizing, presenting a blockchain-empowered framework to manage the lifecycle of AIGC products. Li [36] empirically analyzes the mechanisms of designers' attitudes towards AIGC, and Qi *et al.* [37] explore how AIGC technology influences emotional resonance between users.

Prior literature highlights the role/impact of AIGC or AI innovation. Few works address the generation process of AIGC, including the GAI training, self-learning, smartness enhancement, and the related AIGC offered, thus bringing about the AIGC iteration effect; this paper tries to bridge the gap.

B. AI Marketing

The second category pertains to AI marketing. With the widespread adoption of AI/GAI, researchers have investigated various factors influencing consumers' AI experiences [27]. From the view of generic AI, Khan and Mishra [14] discover that consumers' perception of AI credibility is positively impacted by four factors: data capture, classification, delegation, and social interaction, with perceived justice serving as a mediator. Yang [28] demonstrates that AI service quality and the overall AI experience are perceived through customers' responses to perceived expertise and speed. Additionally, Ortakci and Seker [29] present an AI-based marketing model to profile customer churn and predict a service fee likely accepted by customers. Nevertheless, Wang et al. [30] discover that AI-driven marketing diminishes consumers' motivation to purchase green products. Similarly, Ma et al. [31] find that AI outperforming individuals will trigger a ripple effect, fostering unfavorable consumer attitudes toward AI companies.

Some studies also have been conducted on the aspects of AIGC-marketing, e.g., Wu *et al*. [32], Li and Lee [13], and Bulchand-Gidumal et al. [38]. Among them, Wu *et al*. [32] find that consumers are more likely to share an advertisement placed by AI than an advertisement created by AIGC when performing high-complexity tasks. Li and Lee [13] examine how ChatGPT, as an exemplary representative of AIGC products, influences users' transition from new adopters to loyal advocates and find that these factors include communication quality, personalization, anthropomorphism, and cognitive and emotional trust. Bulchand-Gidumal et al. [38] identify that AIGC transforms customer processes and services in hospitality and tourism marketing by both engaging smart and predictive customer care and employing predictive and augmented service design.

However, no research has explored how AI/AIGC marketing, combined with AIGC innovation, jointly influences the subsequent firm's pricing and product sales. Additionally, prior work merely utilizes empirical rather than optimization methods to examine GAI-based marketing operations.

C. Optimal Control Model in Advertising

A body of optimal-control-advertising models reflects the impact of a firm's advertising effort on a product's market share. Earlier works assume the advertising dynamics to be linear functions [39, 40] or later develop the square-root functions of the market share or independent of the product's goodwill [41, 42, 43]; these models yield optimal advertising strategies under various competitive and/or cooperative environments [44, 45].

Extant studies mainly focus on the interplay between quality, advertising, and goodwill. Among them, Liu et al. [46] study an optimal control model where quality improvement positively affects both goodwill and demand, while marketing influences demand through pricing and advertising efforts. Reddy et al. [47] incorporate quality as a controlling factor in dynamic optimal control advertising models, highlighting the importance of design quality over time. Ni and Li [48] consider product innovation to improve overall quality and explore the impact of quality and advertising on a monopolistic producer's pricing. De Giovanni [49] develops an optimal control model demonstrating the positive effects of advertising and quality on company goodwill, while Zhou and Ye. [50] examine how joint emission reduction affects goodwill and sales within a cooperative advertising strategy framework.

Unlike the above work, we apply the optimal control advertising model to the AIGC settings, where the interplay between GAI smartness level, AI training efforts in advertising, and goodwill are considered. Additionally, the advertising-related knowledge accumulation level is also influenced by AI training efforts in advertising.

In summary, prior literature on AI innovation and AI marketing predominantly concentrates on qualitative and empirical analysis and findings [35]. There is a lack of optimization methodology to investigate two typical AIGC strategies under the AIGC iteration effect, which widely exists in GAI empowering firms' R&D and marketing sectors. We also extend the optimal control advertising model by using the learning-by-doing method to analyze the interplay between GAI smartness level, AI training efforts in advertising, and goodwill.

III. MODEL

This research considers a market where a firm will utilize GAI to enhance its R&D and/or advertising capabilities under two typical options (i.e., the single AIGC and dual AIGC strategies). The optimal-control model and learning-by-doing method are used to establish the GAI analytic framework, where both the AIGC iteration effect and the capital constraint are considered. Next, we proceed to model the single/dual AIGC strategy, respectively.

A. Single AIGC Strategy (AIGC-I): Empowering R&D with GAI

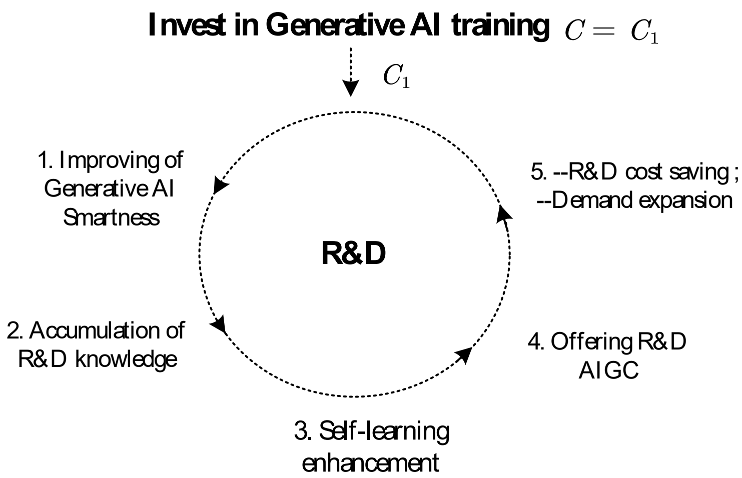

Fig.1. The cyclic process of AIGC iteration under the single AIGC strategy

Under the single AIGC strategy, the firm merely leverages GAI to enhance its R&D capabilities to acquire tailored design solutions (i.e., AIGC). Subsequently, the AIGC-based designs are manufactured as products and targeted at potential consumers. Given the necessity for continuous AIGC upgrades, the firm must train GAI to improve its smartness. The whole cyclic process of AIGC iteration includes the GAI training, GAI smartness improvement, accumulation of R&D knowledge, self-learning enhancement, and offering R&D AIGC, thus bringing about R&D cost-saving and demand expansion for the firm (shown in Fig. 1).

Therefore, following the literature [50], the AIGC-based product demands are determined by GAI smartness level $q(t)$ and selling price $p(t)$, which can be expressed as:

$$D(q(t),p(t)) = b_1 q(t)\,(b_0 - b_3 p(t)), \tag{1}$$

where b_0, b_1, b_3 are constants. Meanwhile, according to the literature [33, 47], the change rate of GAI smartness level can be expressed as Equation (2):

$$\dot{q}(t) = u(t) - \varrho q(t), \tag{2}$$

where $q(t)$ is the GAI smartness level at time *t*, $u(t)$ is the GAI training efforts in the R&D sector at time *t*. Note that the relative GAI smartness level will decay over time (*t*) if there is no training, and assume ϱ is its decay rate.

The firm's GAI training efforts enable GAI to continuously increase the accumulation of relevant R&D knowledge, thus facilitating GAI self-learning and promoting the AIGC iteration. Following the work [33, 34], we assume the process follows an exponential upward trajectory. Thus, the accumulation of related R&D knowledge can be expressed as follows:

$$A_1(t) = e^{-\xi t}\left[A_{10} + \delta \int_0^t e^{\xi t} u(\tau)\, d\tau\right], \tag{3}$$

where A_{10} is the R&D-related knowledge accumulation level at the initial period t_0, $A_1(t)$ is the R&D-related knowledge accumulation level at the period t_1, δ represents the transferring coefficient of knowledge accumulation, and $e^{-\xi t}$ is the discount rate.

Similar to the GAI smartness level, the R&D-related knowledge accumulation level also exists at the decay rate ξ. Based on the work [33, 47], the change rate of the R&D-related

knowledge accumulation level can be expressed through the differential equation as below:

$$\dot{A}_1(t) = \delta u(t) - \xi A_1(t). \tag{4}$$

From the initial period t_0 to the period t_1, the R&D-related knowledge level has accumulated through continuous GAI training, and the corresponding AIGC iteration occurs accordingly, subsequently impacting R&D costs. Following the literature [33, 34], the R&D cost at the period t_1 is illustrated as follows:

$$C_1(u(t), A_1(t)) = \varepsilon u^2(t) - a_1(A_1(t) - A_{10}), \tag{5}$$

where a_1 refers to GAI's self-learning rate regarding R&D knowledge, this means the higher the self-learning rate, the higher the learning-by-doing capabilities of GAI, and the more efficient the firm's AIGC iteration. $A_1(t) - A_{10}$ is the incremental of the R&D knowledge accumulation at the interval $t_1 - t_0$. Therefore, Equation (5) represents that, under the single AIGC strategy, the related R&D cost C_1 decreases with GAI's self-learning capability regarding R&D.

By synthesizing Equations (1) - (5), the firm's profit function under the single AIGC strategy can be developed as follows:

$$\pi(t) = [p(t) - C_0] D(q(t), p(t)) - C_1(u(t), A_1(t)), \tag{6}$$

in which C_0 is the unit production cost.

Therefore, the model under the single AIGC strategy can be expressed as:

$$\begin{gathered} \prod = \max_{u(t), p(t)} \int_0^{\infty} e^{-rt} \{ [p(t) - C_0] D(q(t), p(t)) - C_1(u(t), A_1(t)) \} dt \\ s.t \;\; \dot{q}(t) = u(t) - \varrho q(t) \\ \dot{A}_1(t) = \delta u(t) - \xi A_1(t) \\ C_1(u(t), A_1(t)) = \varepsilon u^2(t) - a_1(A_1(t) - A_{10}) \end{gathered} \tag{7}$$

Note that the total AI training budget is allocated to the R&D sector only, i.e., $C = C_1$.

B. Dual AIGC Strategy (AIGC-II): Empowering Both R&D and Advertising with GAI

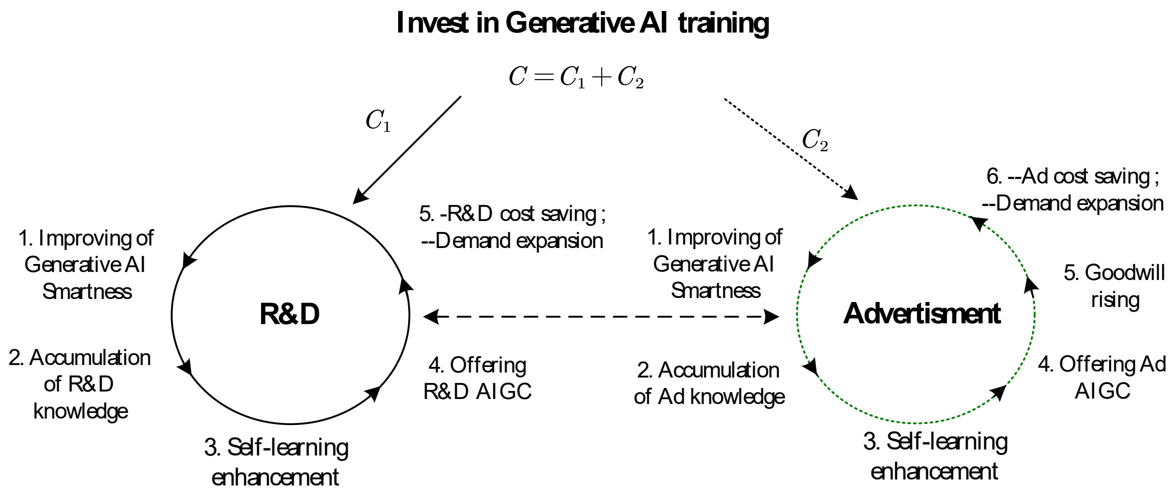

Fig. 2. The cyclic process of dual AIGC iteration

Under the dual AIGC strategy, the firm utilizes GAI to empower both the R&D sector and advertising to obtain AIGC-based R&D solutions and AIGC-based advertising schemes. In order to upgrade both AIGCs, the firm also needs to train GAI to boost its smartness continuously. The whole cyclic process of the dual AIGC iteration is shown in Fig. 2, where it can be seen that the firm first allocates GAI training investment between the R&D sector and the advertising sector under the limited budget, and then carries out two sub-cyclic processes: one is for training R&D-based GAI, while the other for advertising-based GAI, meanwhile two sub-cyclic processes interplay each other.

Both training GAIs are similar to the case of the single AIGC strategy, considering that advertising gives rise to the goodwill, following the work [33, 48, 49], the change rate of the AI smartness level and the firm's goodwill under the dual AIGC strategy can be presented as the below differential equation:

$$\dot{q}(t) = u(t) + k(t) - \varrho q(t),$$

$$\dot{g}(t) = k(t) + \eta q(t) - \beta g(t), \tag{8}$$

where the first expression of Equation (8) represents the change rate of the AI smartness level related to the training efforts in the R&D sector and advertising sector. The second expression of Equation (8) indicates that the change rate of the firm's goodwill is related to the AI training efforts in advertising $k(t)$ and the AI smartness level $q(t)$. In addition, ϱ and β represent the decay rate of AIGC's smartness level and the firm's goodwill, respectively.

As GAI empowers simultaneously in both the R&D and advertising sectors, following the literature [50], thus the product market demand in this context is

$$D(q(t), g(t), p(t)) = (b_1 q(t) + b_2 g(t))\,(b_0 - b_3 p(t)). \tag{9}$$

Similarly, under the dual AIGC strategy, AIGC empowers both the R&D and advertising sectors. GAI experiences the accumulating process of related knowledge over time. That is to say, as the firm continuously makes efforts to train GAI, the related knowledge will unceasingly be collected, thus accelerating the self-learning process of GAI and yielding the AIGC iteration effect with respect to two sectors. Following the work [33, 34, 47], the accumulation of the related advertising knowledge can be expressed as follows:

$$A_2(t) = e^{-\varphi t}\left[A_{20} + \theta \int_0^t e^{\varphi t} k(\tau)\, d\tau\right], \tag{10}$$

$$\dot{A}_2(t) = \theta k(t) - \varphi A_2(t), \tag{11}$$

where A_{20} is the advertising-related knowledge accumulation level at the initial period t_0, $A_2(t)$ is the advertising-related knowledge accumulation level at the period t_1, θ represents the transferring coefficient of the advertising-related knowledge accumulation, and $e^{-\xi t}$ is the discount rate.

Based on the literature [33, 34], considering the AIGC iterative effect, the firm's advertising cost under the dual AIGC strategy at the time *t* can be expressed as:

$$C_2(k(t),A_2(t)) = \phi k^2(t) - a_2(A_2(t) - A_{20}). \quad (12)$$

Equation (12) shows that, from the initial period t_0 to the period t_1, through the continuous training in GAI in the advertising sector, related ad knowledge is accumulated, and the corresponding ad-related AIGC iteration also happens, subsequently impacting advertising costs over time. Here, a_1 refers to the GAI's self-learning rate regarding the advertisement knowledge accumulation.

Similarly, under the dual AIGC strategy, the firm's R&D cost function is the same as under the single AIGC strategy. It is worth noting that under the financial budget constraint, the firm needs to appropriately allocate the expenditures of training GAI between the R&D and advertising sectors, so we have

$$C_1 = xC, \quad (13)$$

$$C_2 = (1-x)C. \quad (14)$$

where *C* represents the whole budget, *x* represents the ratio allocated to training R&D sector, 1-*x* is the ratio allocated to the advertising sector.

Therefore, the total profit function under the dual AIGC strategy during the time *t* is,

$$\pi(t) = [p(t) - C_0]D(q(t),g(t),p(t)) - C_1(u(t),A_1(t)) - C_2(k(t),A_2(t)). \quad (15)$$

To obtain the optimal equilibrium solution by maximizing the profit, the objective function and constraints are,

$$
\prod = \max_{u(t),p(t),k(t)} \int_0^{\infty} e^{-rt} \{ [p(t) - C_0] D(q(t),g(t),p(t)) - C_1(u(t),A_1(t)) - C_2(k(t),A_2(t)) \} dt
$$

$$
\begin{aligned}
s.t \qquad & \dot{q}(t) = u(t) + k(t) - \varrho q(t) \\
& \dot{g}(t) = k(t) + \eta q(t) - \beta g(t) \\
& \dot{A}_1(t) = \delta u(t) - \xi A_1(t) \\
& \dot{A}_2(t) = \theta k(t) - \varphi A_2(t) \\
& C_1(u(t),A_1(t)) = \varepsilon u^2(t) - a_1(A_1(t) - A_{10}) \\
& C_2(k(t),A_2(t)) = \phi k^2(t) - a_2(A_2(t) - A_{20}) \\
& C = C_1 + C_2
\end{aligned} \tag{16}
$$

IV. ANALYSIS AND RESULTS

This section will present the corresponding optimal outcomes and the comparative analysis under the single AIGC strategy and the dual AIGC strategy.

A. Single AIGC Strategy (AIGC-I)

We use the Hamilton function to derive the results from the optimal-control model under the single AIGC strategy, and then the following Propositions and Lemma are shown below (All proofs are shown in the Appendix):

Proposition 1. *Under the single AIGC strategy, i.e., empowering the R&D sector alone, the change rate of the training GAI effort level decreases with the self-learning rate of AIGC knowledge accumulation* ($\frac{\partial \dot{u}(t)}{\partial a_1} < 0$).

Proposition 1 demonstrates that, under the single AIGC strategy, the change rate of AI training effort level decreases with the self-learning rate. This suggests that when GAI self-learning reaches a high level, the requirement for boosting the firm's training efforts gradually wanes. On the contrary, it also means that when GAI development is at an initial stage, the firm should prioritize training GAI rather than relying mainly on its self-learning capability to enhance GAI's smartness level.

Additionally, Proposition 1 also indicates that the impact of R&D-based AIGC empowerment relies on two factors: one is the external factor, i.e., training GAI, and the other is the internal factor, i.e., GAI self-learning ability. These two factors interact and interchange in the firm's GAI evolution, thus implying that the firm needs to intensify the external GAI training efforts before GAI becomes smarter and more capable of R&D work.

Proposition 2. *Under the single AIGC strategy, there exists a steady-state equilibrium in which the optimal training GAI effort level and the optimal smartness level are obtained as below,*

$$\hat{u} = \frac{1}{2\varepsilon(r+\varrho)}\left[\frac{\delta a_1(r+\varrho)}{2\varepsilon(r+\xi)} + \frac{b_1[b_0 - b_3 C_0]^2}{4b_3}\right], \tag{25}$$

$$\hat{q} = \frac{1}{2\varepsilon\varrho(r+\varrho)}\left[\frac{\delta a_1(r+\varrho)}{2\varepsilon(r+\xi)} + \frac{b_1[b_0 - b_3 C_0]^2}{4b_3}\right]. \tag{26}$$

Proposition 2 demonstrates that under a steady-state equilibrium, the firm can achieve relative stability at the optimal level of AI training effort and GAI smartness for a certain duration. This suggests that the firm can precisely assess the extent of external training effort based on the corresponding GAI smartness level. Specifically, suppose both the actual AI smartness and desired levels are known, and then the corresponding training efforts for the R&D sector can be obtained, thus enabling the firm to plan, organize, and operate GAI training programs effectively and efficiently beforehand.

Lemma 1. *Under the single AIGC strategy, the optimal GAI smartness level increases with the AIGC self-learning rate (i.e.,* $\frac{\partial \hat{q}}{\partial a_1} > 0$*).*

Lemma 1 demonstrates that enhancing GAI's self-learning abilities can increase its smartness level. Considering that GAI's self-learning relies on the R&D knowledge accumulation and others (for example, algorithms, computing power, etc.), Lemma 1 also means that when the R&D knowledge accumulation reaches a certain threshold, GAI in the R&D sector with the high self-learning abilities would be eventually like human being to do more complex human jobs in R&D sector, whereas the R&D employees have to exploit the new fields that GAI seldom and never involved before, such as the totally-new product designs and more sophisticated research tasks.

Proposition 3. *Under the single AIGC strategy, the optimal R&D-related knowledge accumulation, demand, and optimal profit are,*

$$\widehat{A}_1 = \frac{\delta}{2\varepsilon\xi(r+\varrho)}\left[\frac{\delta a_1(r+\varrho)}{2\varepsilon(r+\xi)} - \frac{b_1[b_0-b_3C_0]^2}{4b_3}\right], \tag{27}$$

$$\widehat{D} = \frac{b_1(b_0-b_3C_0)}{4\varepsilon\varrho(r+\varrho)}\left[\frac{\delta a_1(r+\varrho)}{2\varepsilon(r+\xi)} + \frac{b_1[b_0-b_3C_0]^2}{4b_3}\right], \tag{28}$$

$$\pi = \frac{b_1\Phi(b_0-b_3C_0)^2}{8b_3\varepsilon\varrho\Delta} - \frac{\Phi^2}{4\varepsilon\Delta^2} + a_1\left(\frac{\delta\Phi}{2\varepsilon\xi\Delta} - A_{10}\right), \tag{29}$$

where $\Phi = \dfrac{\delta a_1(r+\varrho)}{2\varepsilon(r+\xi)} + \dfrac{b_1[b_0-b_3C_0]^2}{4b_3}$, $\Delta = \varepsilon(r+\varrho)$.

B. Dual AIGC Strategy (AIGC-II)

To solve the decision problem under the dual AIGC strategy using the Hamilton function, we can obtain the following propositions (All proofs are shown in the Appendix).

Proposition 4. *Under the dual AIGC strategy, we have* (1) $\frac{\partial \dot{u}(t)}{\partial k(t)} < 0$, $\frac{\partial \dot{k}(t)}{\partial u(t)} < 0$; (2) $\frac{\partial \dot{u}(t)}{\partial a_1} < 0$, $\frac{\partial \dot{k}(t)}{\partial a_2} < 0$.

Proposition 4 (1) demonstrates that, under the dual AIGC strategy, the change rate of GAI training efforts in the R&D sector decreases with the change rate of GAI training efforts in the advertising sector. Likewise, the change rate of GAI training efforts in the advertising sector decreases with the GAI training efforts in the R&D sector. That is to say, more efforts or resources allocated in one sector inevitably lead to a decrease in another. This phenomenon displays an "opposing effect" regarding the firm's training effort allocation between the R&D and advertising sectors. It suggests that engineering managers need to know which sector is their priority for adopting GAI empowerment, and then allocate more resources.

Proposition 4(2) illustrates that, under the dual AIGC strategy, the change rates of AI training efforts for both R&D and advertising sectors diminish with their respective self-learning rates. Similar to Proposition 1, it further proves that when the firm's GAI self-learning capabilities in both sectors are at higher levels, the role of GAI training for R&D and advertising sectors slowly attenuates in improving GAI smartness.

Proposition 5. *Under the dual AIGC strategy, the steady-state equilibrium exists, i.e.,*

$$\hat{u} = \frac{\delta M}{2\varepsilon M(r+\xi)} a_1 + \left[\frac{(r+\beta)M+\eta}{M^2(r+\beta)-\eta} + \frac{1}{M}\right] \frac{\theta\eta a_2}{2\varepsilon(r+\varphi)} + E,$$

$$\hat{k} = \frac{1}{\phi[M(r+\beta)-\eta]} \left[\frac{\theta[(r+\beta)M+\eta]}{2(r+\varphi)} a_2 + L\right],$$

$$\hat{q} = \frac{\delta M}{2\varepsilon\varrho M(r+\xi)} a_1 + \frac{(T+MS)\theta\eta\phi + \theta\varepsilon T M^2}{2\varepsilon\varrho\phi M^2 S(r+\varphi)} a_2 + F, \text{ and}$$

$$\hat{g} = \frac{\delta M}{2\varepsilon\varrho M(r+\xi)}a_1 + \left[\frac{\theta S}{2\phi\beta(r+\varphi)T} + \frac{(T+MS)\theta\eta\phi + \theta\varepsilon T M^2}{2\varepsilon\varrho\phi M^2 S(r+\varphi)}\right]a_2 + \frac{L}{\phi\beta T} + F,$$

where

$$E = \frac{\eta[b_2 M - b_1](b_0 - b_3 C_0)^2}{8b_3\varepsilon[M^2(r+\beta)-\eta]} - \frac{b_1(b_0 - b_3 C_0)^2}{42\varepsilon b_3 M}, \quad M = r + \varrho + \eta,$$

$$L = \frac{[b_2 M - b_1](b_0 - b_3 C_0)^2}{8b_3}, \quad S = (r+\beta)M + \eta, \quad T = M(r+\beta) - \eta,$$

$$F = E + \frac{[b_2 M - b_1](b_0 - b_3 C_0)^2}{8\phi b_3[M(r+\beta)-\eta]}.$$

Proposition 6. *Under the dual AIGC strategy, we have (1)* $\frac{\partial \hat{u}}{\partial a_1} > 0$, $\frac{\partial \hat{u}}{\partial a_2} > 0$; *(2)* $\frac{\partial \hat{k}}{\partial a_2} > 0$, $\frac{\partial \hat{k}}{\partial a_1} = 0$.

Proposition 6(1) demonstrates that, under the dual AIGC strategy, the AI training effort level for R&D increases with its self-learning rate in R&D under steady-state equilibrium, and the self-learning rate in advertising positively influences the level of AI training effort for R&D. Proposition 6(2) shows that the AI training effort level for advertising increases with its self-learning rate in advertising. Surprisingly, the self-learning rate in R&D does not affect the level of AI training effort for advertising.

This phenomenon indicates that the rate of self-learning in the upstream sector has a “spillover effect,” while the downstream sector’s self-learning rate does not show the same pattern. Specifically, the self-learning rate in the downstream sector only has a positive impact on its own advertising sector and does not extend to the upstream R&D sector. This counter-intuitive result underscores the significance of GAI empowering the upstream R&D sector is more important than the downstream advertising sector, thus implying that firms should prioritize the investment of GAI in the R&D sector rather than the advertising sector because the adopting of R&D-based GAI has the spillover effect on other sectors including advertising sector.

Proposition 7. *Under the dual AIGC strategy, we have (1)* $\frac{\partial \hat{q}}{\partial a_1} > 0$, $\frac{\partial \hat{q}}{\partial a_2} > 0$; *(2)* $\frac{\partial \hat{g}}{\partial a_1} > 0$, $\frac{\partial \hat{g}}{\partial a_2} > 0$.

Proposition 7 delineates that, at the steady-state equilibrium, the level of AI smartness increases with both R&D and advertising self-learning rates, and the firm's goodwill also rises with both R&D and advertising self-learning rates. These findings suggest that both self-learning rates positively influence not only the smartness level but also the goodwill of the firm. Unlike Proposition 6, where the impact was solely on the upstream R&D effort level, both self-learning rates in Proposition 7 positively affect the "smartness level" and "goodwill" of the firm. It reveals that the R&D-empowered GAI has a positive indirect impact on enhancing the firm's goodwill, which the advertising-empowered GAI aims to focus on. It implies that the dual AIGC strategy can obtain the result of "1+1>2". This synergy effect suggests that adopting the dual AIGC strategy is better off than adopting the single AIGC strategy.

V. NUMERICAL ANALYSIS

To further scrutinize the profit, demand, and unit cost contribution disparities between the single AIGC strategy and the dual AIGC strategy, we will use numerical studies to compare them under capital constraints. To ensure the case replicability and reliability, we first surveyed 78 middle/high-class managers to write down the most typical five AIGC industrial applications in the GAI Innovation and GAI advertising sectors. Thus, we acquired the result that the automobile, electronics, electrical appliances, machinery, and textile clothing fields are ranked in the top 5. The outcomes are consistent with reality. Therefore, we employ the case study of AITO, a renowned Chinese electric vehicle (EV) brand produced by EV manufacturer *Selis*, which adopted *Huawei*'s GAI---Pangu.

In this case, *Selis* initially adopted *Huawei*'s Pangu GAI to empower its R&D sector for its M5 product. Subsequently, facing fierce competition in the EV market, *Selis* extended the application of *Huawei*'s Pangu GAI to the advertising sector in the subsequent M7 and M9 products. To guarantee the reliability of the data collected, we first collected the relevant data based on the annual report disclosed by *Selis*. On the other hand, we obtained the relevant online data through web crawlers. Then, through the on-site interviews with executives from *Huawei* and *Selis*, we supplemented the missing data. Hence, we gathered the data outlined in Table 1.

Table 1. Parameters of Numerical Analysis

Parameters	b_0	b_1	b_2	b_3	η	δ	ξ	A_{10}	A_{20}	g_0
Values	5	0. 2	0. 2	1	0. 2	0. 2	0. 2	1	1	0. 2
Parameters	C_0	θ	φ	ϱ	β	r	ε	ϕ	q_0	a_2
Values	0. 5	0. 2	0. 2	0. 2	0. 2	0. 1	0. 5	0. 5	0. 2	4

A. Impact of R&D Self-Learning Rate on Profits under Two AIGC Strategies

Fig.3 illustrates that the R&D self-learning rate positively correlates with the profits under both strategies. However, the profit generated under the dual AIGC strategy significantly surpasses that under the single AIGC strategy. This shows the advantages of empowering both the R&D and advertising sectors over empowering only one sector via GAI. Furthermore, under the dual AIGC strategy, the conversion of AIGC empowerment into firm profits is notably more apparent. This observation aligns with the strategy adopted by *Selis* EV brand AITO, which initially adopted R&D-based AIGC from *Huawei*'s GAI and later expanded to the advertising AIGC. Consequently, AITO's M7 and M9 products gained more popularity in China's EV market than others. Therefore, it suggests that when firms leverage GAI, they should prioritize empowering both sectors because the effectiveness of AIGC empowering one sector is most likely to be discounted compared to the multiple ones in terms of profitability.

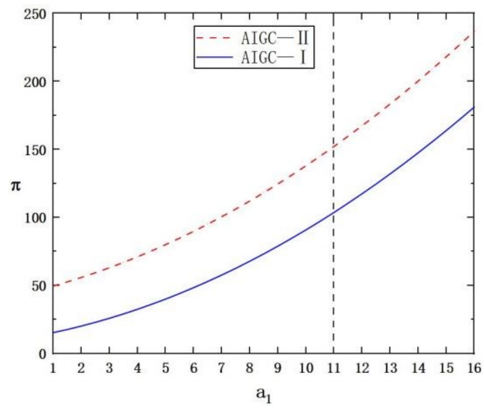

Fig. 3. The relationship between R&D self-learning rate and profit

B. Impact of Self-Learning Rates on Profit and Demand under Dual AIGC Strategy

Considering the distinctions between the R&D self-learning rate and advertising self-learning rate under dual AIGC strategy, Fig. 4 and 5 illustrate the joint influence of two factors on the firm's profit and demand, respectively.

Fig. 4 and 5 show that the firm's profit and demand increase with the AI R&D self-learning rate and the AI advertising self-learning rate. Notably, both self-learning rates exert a more significant influence on profits than demand. Furthermore, the effects of these two types of AI self-learning rates on profit are relatively modest in the early stage but become more pronounced later. However, regarding the demand, two AI self-learning rates demonstrate a more panel-shaped trend across both stages. This indicates that, as time goes by, the firm's profits from using GAI become increasingly significant. It implies that, regardless of whether in the marketing sector or R&D sector, the earlier firms use GAI, the more self-learning capabilities are continuously improved. Therefore, firms should apply GAI sooner rather than later.

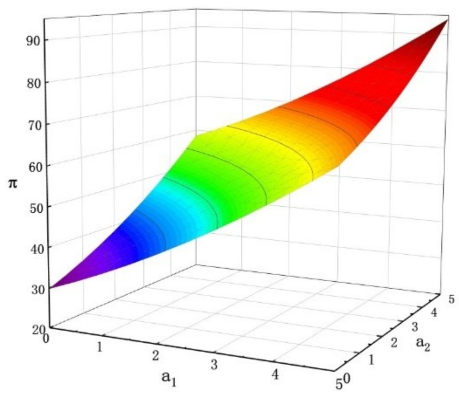

Fig. 4. The impact of both self-learning rates on profits

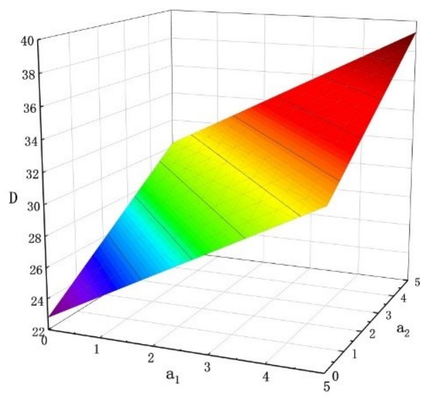

Fig. 5. The impact of both self-learning rates on demand

C. Contribution Rate of Unit Cost under Two Different AIGC Strategies

To assess the efficacy of investing in AI training within the R&D and advertising sectors under the limited budget, we utilize the metric $q(t)/C_1$ to measure the unit training-cost contribution to the smartness level under two different AIGC strategies. Fig. 6 illustrates that, under two AIGC strategies, the contribution rate of the AI smartness level exhibits an upward trajectory. Notably, under the single AIGC strategy, this rate surpasses that under the dual AIGC strategy. Particularly, when $a_1 > 4$, the discrepancy in contribution rate becomes more pronounced. This suggests that higher investments in AI training for the R&D sector leads to a swifter enhancement in the AI smartness level, resulting in more significant and sustainable benefits for the firm. It also underscores early adopters' critical role in the R&D sector for the future development of firms.

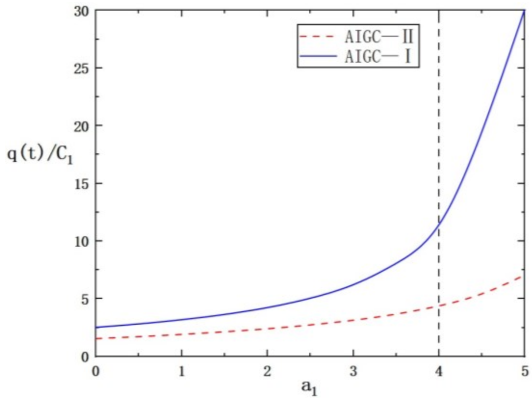

Fig. 6. Comparison of $q(t)/C_1$ between two different AIGC strategies

D. Contribution Rate of Unit Cost Under Dual AIGC Strategy

Under dual AIGC strategy, Fig. 7 uses the metric $g(t)/C_2$ and $q(t)/C_1$ to measure the unit training cost contribution to the firm's goodwill and the GAI smartness level, respectively. It demonstrates that the contribution to goodwill from the advertising sector exceeds the contribution to the smartness level from the R&D sector. This discrepancy reveals why many firms initially opt for GAI empowerment toward the downstream advertising sector instead of the upstream R&D sector. The rationale is that the firm's GAI training in the R&D sector shows more challenges than in the advertising sector. This also implies that the firm should not merely focus on the advertising sector in the short term, and neglect to invest in the R&D sector for long-term development. That is to say, the firm should carefully balance the capital allocation between two sectors.

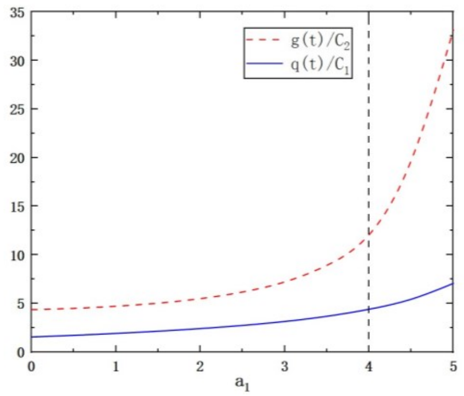

Fig. 7. Comparison between $q(t)/C_1$ and $g(t)/C_2$ under dual AIGC strategy

VI. EXTENSION

The above analysis is conducted based on the multiplicatively separable function; this form is a relatively simple and general way to model the relationship between price, GAI smart level, and the firm's goodwill. Next, this section will further examine an additively separable demand function to observe the related outcomes for checking the robustness. Given demand functions capturing the effects of GAI smart level, the firm's goodwill and price under the single AIGC strategy and the dual AIGC strategy; (1) and (9) become

$$\dot{D}(q(t),p(t)) = b_0 - b_3 p(t) + b_1 q(t) \qquad (1A)$$

$$\ddot{D}(q(t),g(t),p(t)) = b_0 + b_1 q(t) + b_2 g(t) - b_3 p(t) \qquad (9A)$$

Substituting (1A) and (9A) in (7) and (16), respectively, the following Propositions can be obtained by using the previous method.

Proposition 8. *For an additively separable demand function, the optimal GAI smartness level under the single AIGC strategy increases with the AIGC self-learning rate (i.e.,* $\frac{\partial \dot{u}(t)}{\partial a_1} < 0$*).*

Proposition 8 exhibits that, for an additively separable demand function, the change rate of AI training effort level under the single AIGC strategy decreases with the self-learning rate. The outcome is similar to the multiplicatively separable function, which further proves that when GAI development is not at a mature stage, the firm should prioritize training GAI rather than mainly relying on its self-learning capability to enhance GAI's smartness level.

Proposition 9. *For an additively separable demand function, given the R&D self-learning rate, the profit under the dual AIGC strategy exceeds that under the single AIGC strategy.*

Proposition 9 also displays that, for an additively separable demand function, the advantages of empowering both the R&D and advertising sectors over empowering only one sector via GAI, which indicates that, given a certain R&D self-learning rate, the dual AIGC strategy is better off than the single AIGC strategy without capital constrain owing to the synergy effect.

Ⅶ. DISCUSSION AND LIMITATION

A. Theoretical Contributions

This paper has the following three contributions to the engineering management literature. First, this work contributes to a novel GAI-related analytic framework for engineering managers. Previous work mainly focuses on the role and impact of GAI by the empirical and static analysis method [9, 10, 13, 14, 23]; our research goes a big step forward in this domain by utilizing the optimal control and learning-by-doing method in consideration of the AIGC iteration process, from GAI training to knowledge accumulation to GAI self-learning to AIGC update, and then dynamically present the multi-periodic AIGC strategy models.

Second, this paper offers two typical AIGC strategies that engineering managers are most concerned about and explores their differences. Although Liu *et al*. [25] present a blockchain-based AIGC strategy for empowering the R&D sector to protect the AIGC from tampering and plagiarization, and Wu *et al.* [32] empirically compare the placed-by-AI and created-by-AI (AIGC) strategies in high-complexity tasks, we extend this research line from one generic AIGC strategy to two typical AIGC strategies, namely, the single AIGC and the dual AIGC strategy.

Third, this paper also advances the research on GAI empowerment from one sector to across different sectors (i.e., R&D and advertising/marketing sectors), and explores their interaction. Chesterman [24] and Li [26] only study AI empowerment in the R&D sector, whereas Li and Lee [13] and Wu *et al*. [32] merely investigate AI empowerment in the marketing sector. In this research, we broaden the scope of the AI analytic framework by presenting the across-sector strategy (i.e., the dual AIGC strategy) to offer a more realistic picture of the firm's GAI development for engineering managers.

B. Results and Practical Implications

Our findings bear the following main results and practical implications.

First, our findings show that, under the dual AIGC strategy, the self-learning rate in advertising positively influences the level of AI training effort for R&D; surprisingly, the self-learning rate in R&D does not affect the level of AI training effort for advertising. This counter-intuitive result indicates that the rate of self-learning in the upstream sector has a "spillover effect," while the downstream sector's self-learning rate does not show the same pattern. Specifically, the self-learning rate in the downstream sector only positively impacts its own advertising sector and does not extend to the upstream R&D sector. It suggests the significance of GAI empowering the upstream R&D sector is more important than the downstream advertising sector, thus implying that engineering managers should prioritize the GAI in the R&D sector instead of the advertising sector because the adopting of R&D-based GAI has the spillover effect on other sectors including advertising sector.

Second, the outcome shows that, under the single AIGC strategy, the change rate of AI training effort level in the R&D sector decreases with its self-learning rate. This uncovers that considering the AIGC iteration effect, there are two crucial factors of impact on R&D-based AIGC empowerment: one is the external factor, i.e., training GAI, and the other is the internal factor, i.e., GAI self-learning ability. These two factors interact and interchange in the firm's GAI evolution, thus implying that the firm needs to intensify the external GAI training efforts before GAI becomes smarter and more capable of R&D work.

Third, this research displays that the level of AI smartness and the firm's goodwill increases with both R&D and advertising self-learning rates, which reveals that R&D-empowered GAI has an indirect and positive impact on enhancing the firm's goodwill that the advertising-empowered GAI aims to focus on. It suggests to engineering managers that the dual AIGC strategy can obtain the result of "1+1>2"; this synergy effect implies that adopting the dual AIGC strategy is better off than adopting the single AIGC strategy.

Fourth, we also find that the investment contribution rate of AI smartness under the single AIGC strategy surpasses that under the dual AIGC strategy. This suggests that focusing on investments in the AI training R&D sector alone can enhance the AI smartness level more than adopting the dual AIGC strategy. Meanwhile, if the dual AIGC strategy is adopted, the investment contribution to goodwill from the advertising sector exceeds the contribution to the smartness level of the R&D sector. This discrepancy reveals why many firms initially opt for GAI empowerment toward the downstream advertising sector instead of the upstream R&D sector. The rationale is that the firm's GAI training in the R&D sector shows more challenges than in the advertising sector. This suggests that engineering managers should not merely focus on the advertising sector in the short term. That is to say, the firm should carefully balance the capital allocation between two sectors under the dual AIGC strategy for long-term development.

C. Limitations and Future Research

The study has several limitations that offer avenues for future research. Firstly, the model is confined to the scenarios where AIGC empowers only the R&D and advertising sectors, neglecting other sectors like manufacturing and logistics. Future studies could broaden the scope to include these sectors, providing a more comprehensive understanding of AIGC's impact across the entire supply chain. Secondly, the analysis focuses on a single firm adopting AIGC powered by GAI, overlooking the competitive dynamics when multiple firms employ AIGC strategies. Future research could explore these dynamics by examining how two competitive firms utilize AIGC strategies and their effects on their performance. Additionally, this study does not delve into the implications of AIGC strategies in the supply chain setting. Exploring the interactions between supply chain management could offer valuable insights into optimizing supply chain operations.

Ⅷ. CONCLUSION

This paper explores the role of GAI in empowering a firm's R&D capabilities, subsequently extending its utility to advertising through AIGC. During the AIGC empowerment process, the AIGC iteration is paramount, requiring continuous GAI training and self-learning. To address GAI-related operations under AIGC iteration effect, we employ the optimal control and learning-by-doing methods, offering a nuanced analysis and comparison of two predominant AIGC strategies. The results provide some useful and practical insights for engineering managers to improve firms' competitiveness in the era of GAI.

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APPENDIX

A. ***Proof of Proposition 1***.

At the interval $t \in [0, +\infty)$, to obtain the optimal training GAI effort level $u(t)$, selling price $p(t)$, and optimal AI smartness level $q(t)$, following the work [46, 47, 48], we have the Hamilton function to optimize the above models. The Hamilton function can be achieved from Equation (7):

$$
\begin{aligned}
H = &[p(t) - C_0][b_1 q(t)][b_0 - b_3 p(t)] - [\varepsilon u^2(t) - a_1 (A_1(t) - A_{10})] \\
&+ \lambda_1 [u(t) - \varrho q(t)] + \lambda_2 [\delta u(t) - \xi A_1(t)]
\end{aligned}
, \tag{17}
$$

where λ_1, λ_2 represent the dynamic adjoint variables, and the corresponding necessary conditions are:

$$
\frac{\partial H}{\partial u(t)} = -2\varepsilon u(t) + \lambda_1(t) + \delta\lambda_2(t) = 0, \tag{18}
$$

$$
\frac{\partial H}{\partial p(t)} = b_1 q(t)[b_0 + b_3 C_0 - 2b_3 p(t)] = 0. \tag{19}
$$

The corresponding co-state equations obtained are:

$$
\dot{\lambda}_1(t) = r\lambda_1 - \frac{\partial H}{\partial q(t)} = (r + \varrho)\lambda_1(t) - b_1[p(t) - C_0][b_0 - b_3 p(t)], \tag{20}
$$

$$
\dot{\lambda}_2(t) = r\lambda_2(t) - \frac{\partial H}{\partial A_1(t)} = (r + \xi)\lambda_2(t) - a_1. \tag{21}
$$

From the equation (18), we can obtain that $2\varepsilon\dot{u}(t) = \dot{\lambda}_1(t) + \delta\dot{\lambda}_2(t)$ (18A)

The boundary conditions should be met as $\lim_{t\to+\infty} e^{-rt}\lambda_1(t) = 0$ and $\lim_{t\to+\infty} e^{-rt}\lambda_2(t) = 0$, with these boundary conditions, the lateral condition can be obtained as $\lim_{t\to+\infty} e^{-rt}u(t) = 0$.

According to the price expression Equation (19), the optimal selling price under the single AIGC strategy is obtained as follows:

$$
p(t) = \frac{b_0 + b_3 C_0}{2b_3}. \tag{22}
$$

By solving the differential Equation (21) and utilizing the boundary conditions, we can obtain,

$$
\lambda_2(t) = \frac{a_1}{r + \xi}. \tag{23}
$$

Meanwhile, from Equation (18), we can obtain $\lambda_1(t) = 2\varepsilon u(t) - \delta\lambda_2(t)$, and by substituting Equation (23), we can obtain:

$$\lambda_1(t) = 2\varepsilon u(t) - \frac{\delta a_1}{r+\xi}. \tag{24}$$

Substituting equations (20) and (21) into equation (18A) yields:

$$2\varepsilon\dot{u}(t) = 2\varepsilon(r+\varrho)u(t) - b_1[p(t) - C_0][b_0 - b_3 p(t)] \tag{18B}$$

From equation (22), it is known $p(t) = \frac{b_0 + b_3 C_0}{2b_3}$, substituting it into the above formula (18B) and simplifying it, we can obtain:

$$\dot{u}(t) = (r+\varrho)u(t) - \frac{\delta a_1(r+\varrho)}{2\varepsilon(r+\xi)} - \frac{b_1[b_0 - b_3 C_0]^2}{8\varepsilon b_3} \tag{16}$$

Take the first derivative of formula (16), and thus Proposition 1 is obtained.

B. Proof of Proposition 2.

Under steady-state conditions, the rate of change in the effort level of the enterprise in empowering the R&D process with the GAI model remains unchanged. Therefore, there exists a solution. Substituting it into Equation (16), it can be obtained that $(r+\varrho)u(t) - \frac{\delta a_1(r+\varrho)}{2\varepsilon(r+\xi)} - \frac{b_1[b_0 - b_3 C_0]^2}{8\varepsilon b_3} = 0$, and therefore, the effort level of the enterprise's AIGC empowering the R&D process can be obtained as

$$\hat{u} = \frac{1}{2\varepsilon(r+\varrho)}\left[\frac{\delta a_1(r+\varrho)}{2\varepsilon(r+\xi)} + \frac{b_1[b_0 - b_3 C_0]^2}{4b_3}\right].$$

Similarly, it can be concluded $\hat{q} = \frac{1}{2\varepsilon\varrho(r+\varrho)}\left[\frac{\delta a_1(r+\varrho)}{2\varepsilon(r+\xi)} + \frac{b_1[b_0 - b_3 C_0]^2}{4b_3}\right]$, the proposition can be proven.

C. Proof of Lemma 1.

According to formula (18), we have $\dfrac{\partial \hat{q}}{\partial a_1} = \dfrac{\delta (r+\varrho)}{4\varrho\varepsilon^2 (r+\varrho)(r+\xi)} > 0$, Lemma 1 is proven.

D. ***Proof of Proposition 3.***

Based on the formula of enterprise knowledge accumulation $\dot{A}_1(t) = \delta u(t) - \xi A_1(t)$ combined with the formula $\hat{u} = \dfrac{1}{2\varepsilon(r+\varrho)}\left[\dfrac{\delta a_1(r+\varrho)}{2\varepsilon(r+\xi)} + \dfrac{b_1[b_0 - b_3 C_0]^2}{4b_3}\right]$, it can be concluded that the knowledge accumulation under the R&D stage empowered by the single AIGC strategy of the enterprise is $\hat{A}_1 = \dfrac{\delta}{2\varepsilon\xi(r+\varrho)}\left[\dfrac{\delta a_1(r+\varrho)}{2\varepsilon(r+\xi)} - \dfrac{b_1[b_0 - b_3 C_0]^2}{4b_3}\right]$.

Similarly, due to the demand function of the market, combined with Equations (13) and (18), the market demand of the enterprise can be obtained as

$$\hat{D} = \frac{b_1(b_0 - b_3 C_0)}{4\varepsilon\varrho(r+\varrho)}\left[\frac{\delta a_1(r+\varrho)}{2\varepsilon(r+\xi)} + \frac{b_1[b_0 - b_3 C_0]^2}{4b_3}\right].$$

Based on the profit function empowered by enterprise AIGC $\pi(t) = [p(t) - C_0]D(q(t), p(t)) - C_1(u(t), A_1(t))$, combined with the optimal selling price of the product $\hat{p}$, market demand $\hat{D}$, training effort level $\hat{u}$, and the R&D knowledge accumulation value of the AI big model $\hat{A}_1$, the optimal profit of the enterprise is

$$\pi = \frac{b_1 \Phi (b_0 - b_3 C_0)^2}{8 b_3 \varepsilon \varrho \Delta} - \frac{\Phi^2}{4\varepsilon \Delta^2} + a_1 \left(\frac{\delta \Phi}{2\varepsilon\xi\Delta} - A_{10} \right) .$$

E. Proof of Proposition 4.

Following the work [46, 47, 48], we have the Hamiltonian function (30):

$$\begin{aligned} H=&[p(t)-C_0][b_1 q(t)+b_2 g(t)][b_0-b_3 p(t)]-[\varepsilon u^2(t)-a_1(A_1(t)-A_{10})] \\ &-[\phi k^2(t)-a_2(A_2(t)-A_{20})]+\lambda_3[u(t)+k(t)-\varrho q(t)]+\lambda_4[\delta u(t)-\xi A_2(t)], \\ &+\lambda_5[k(t)+\eta q(t)-\beta g(t)]+\lambda_6[\theta k(t)-\varphi A_3(t)] \end{aligned} \tag{30}$$

where λ_3, λ_4, λ_5, and λ_6 represent the dynamic adjoint variables, and the corresponding necessary conditions are:

$$\frac{\partial H}{\partial u(t)}=-2\varepsilon u(t)+\lambda_3(t)+\delta\lambda_4(t)=0, \tag{31}$$

$$\frac{\partial H}{\partial k(t)}=-2\phi k(t)+\lambda_5(t)+\lambda_3(t)+\theta\lambda_6(t)=0, \tag{32}$$

$$\frac{\partial H}{\partial p(t)}=[b_1 q(t)+b_2 g(t)][b_0+b_3 C_0-2b_3 p(t)]=0, \tag{33}$$

$$\dot{\lambda}_3(t)=r\lambda_3-\frac{\partial H}{\partial q(t)}=(r+\varrho)\lambda_3(t)-b_1[p(t)-C_0][b_0-b_3 p(t)]-\eta\lambda_5(t), \tag{34}$$

$$\dot{\lambda}_4(t)=r\lambda_4(t)-\frac{\partial H}{\partial A_1(t)}=(r+\xi)\lambda_4(t)-a_1, \tag{35}$$

$$\dot{\lambda}_5(t)=r\lambda_5(t)-\frac{\partial H}{\partial g(t)}=(r+\beta)\lambda_5(t)-b_2[p(t)-C_0][b_0-b_3 p(t)], \tag{36}$$

$$\dot{\lambda}_6(t)=r\lambda_6(t)-\frac{\partial H}{\partial A_2(t)}=(r+\varphi)\lambda_6(t)-a_2. \tag{37}$$

By solving the above equations, the optimal effort levels in training GAI with respect to the R&D and advertising sectors, $u(t), k(t)$, $(t\in[0,+\infty))$, selling price $p(t)$, and the AI smartness level $q(t)$ under the dual AIGC strategy can be obtained.

According to Equation (33), the optimal price can be derived as follows:

$$p(t)=\frac{b_0+b_3 C_0}{2b_3}. \tag{38}$$

To satisfy the terminal boundary conditions: $\lim_{t\to+\infty} e^{-rt}\lambda_3(t)=0$, $\lim_{t\to+\infty} e^{-rt}\lambda_4(t)=0$, $\lim_{t\to+\infty} e^{-rt}\lambda_5(t)=0$, and $\lim_{t\to+\infty} e^{-rt}\lambda_6(t)=0$, the terminal lateral conditions can be obtained as $\lim_{t\to+\infty} e^{-rt}u(t)=0$ and $\lim_{t\to+\infty} e^{-rt}k(t)=0$.

By solving differential Equations (34) and (36) and utilizing terminal boundary conditions, we obtain:

$$\lambda_4(t)=\frac{a_1}{r+\xi}\ , \tag{39}$$

$$\lambda_6(t)=\frac{a_2}{r+\varphi}. \tag{40}$$

Substitute Equations (39) and (40) into Equations (31) and (32), respectively, we derive the following equations:

$$\lambda_3(t)=2\varepsilon u(t)-\frac{\delta a_1}{r+\xi}, \tag{41}$$

$$\lambda_5(t)=2\phi k(t)-2\varepsilon\mu(t)-\frac{\theta a_2}{r+\varphi}+\frac{\delta a_1}{r+\xi}. \tag{42}$$

Using equation $\lambda_3(t)=2\varepsilon u(t)-\frac{\delta a_1}{r+\xi}$, and substituting the equation $\lambda_5(t)=2\phi k(t)-2\varepsilon\mu(t)-\frac{\theta a_2}{r+\varphi}+\frac{\delta a_1}{r+\xi}$ into Equation (35)

$$\dot{\lambda}_3(t)=r\lambda_3-\frac{\partial H}{\partial q(t)}=(r+\varrho)\lambda_3(t)-b_1[p(t)-C_0][b_0-b_3p(t)]-\eta\lambda_5(t).$$

Using the same way as Proposition 1, we obtain the following equation,

$$\begin{aligned}2\varepsilon\dot{u}(t)=&(r+\varrho)\left[2\varepsilon u(t)-\frac{\delta a_1}{r+\xi}\right]-b_1[p(t)-C_0][b_0-b_3p(t)]\\&-\eta\left[2\phi k(t)-2\varepsilon\mu(t)-\frac{\theta a_2}{r+\varphi}+\frac{\delta a_1}{r+\xi}\right]\end{aligned}$$

Substitute Equation (38) $p(t)=\frac{b_0+b_3C_0}{2b_3}$ and simplify it, we have

$$\dot{u}(t)=(r+\varrho+\eta)u(t)-\frac{\phi\eta}{\varepsilon}k(t)-\frac{1}{2\varepsilon}\left[\frac{a_1\delta(r+\varrho+\eta)}{r+\xi}+\frac{a_2\theta\eta}{r+\varphi}-\frac{b_1(b_0-b_3C_0)^2}{4b_3}\right]. \tag{43}$$

$$2\phi\dot{k}(t)=(r+\beta)\left[2\phi k(t)-2\varepsilon\mu(t)-\frac{\theta a_2}{r+\varphi}+\frac{\delta a_1}{r+\xi}\right].$$
$$-b_2[p(t)-C_0][b_0-b_3p(t)]$$

After simplification, we have

$$\dot{k}(t)=(r+\beta)k(t)-\frac{1}{2\phi}\left[2\varepsilon\mu(t)+\frac{a_2\theta(r+\beta)}{r+\varphi}-\frac{\delta a_1}{r+\xi}+\frac{b_2(b_0-b_3C_0)^2}{4b_3}\right]. \tag{44}$$

By using formulas (43) and (44), we can obtain:

$$\frac{\partial\dot{u}(t)}{\partial k}=-\frac{\phi\eta}{\varepsilon}<0,\ \frac{\partial\dot{k}(t)}{\partial u}=-\frac{\varepsilon}{\phi}<0$$

$\frac{\partial\dot{u}(t)}{\partial a_1}=-\frac{1}{2\varepsilon}\frac{\delta(r+\varrho+\eta)}{r+\xi}<0,\ \frac{\partial\dot{k}(t)}{\partial a_2}=-\frac{1}{2\phi}\frac{\theta(r+\beta)}{r+\varphi}<0$, and Proposition 4 is proven.

F. Proof of Proposition 5.

Under steady-state conditions, the rate of change in the research and development efforts of enterprises in training AIGC remains unchanged, while the rate of change in the effort of enterprises in training AIGC for goodwill remains unchanged. Therefore, there exists $\dot{u}(t)=0$, $\dot{k}(t)=0$, and substituting equations (43) and (44) yields:

$$(r+\varrho+\eta)u(t)-\frac{\phi\eta}{\varepsilon}k(t)-\frac{1}{2\varepsilon}\left[\frac{a_1\delta(r+\varrho+\eta)}{r+\xi}+\frac{a_2\theta\eta}{r+\varphi}-\frac{b_1(b_0-b_3C_0)^2}{4b_3}\right]=0,$$

$$(r+\beta)k(t)-\frac{\varepsilon}{\phi}\mu(t)-\frac{1}{2\phi}\left[\frac{a_2\theta(r+\beta)}{r+\varphi}-\frac{\delta a_1}{r+\xi}+\frac{b_2(b_0-b_3C_0)^2}{4b_3}\right]=0,$$

Therefore, the optimal goodwill AIGC effort level and the training AIGC R&D effort level are obtained as follows:

$$\hat{k}=\frac{1}{\phi[M(r+\beta)-\eta]}\left[\frac{\theta[(r+\beta)M+\eta]}{2(r+\varphi)}a_2+L\right]$$

$$\hat{u}=\frac{\delta M}{2\varepsilon M(r+\xi)}a_1+\left[\frac{(r+\beta)M+\eta}{M^2(r+\beta)-\eta}+\frac{1}{M}\right]\frac{\theta\eta a_2}{2\varepsilon(r+\varphi)}+E$$

According to the formula $\dot{q}(t)=u(t)+k(t)-\varrho q(t)$, $\dot{g}(t)=k(t)+\eta q(t)-\beta g(t)$, under steady state conditions, $\dot{q}(t)=0$, $\dot{g}(t)=0$, and by substituting $\hat{k}$, $\hat{u}$, the optimal R&D smartness level and optimal goodwill can be obtained:

$$\hat{q}=\frac{\delta M}{2\varepsilon\varrho M(r+\xi)}a_1+\frac{(T+MS)\theta\eta\phi+\theta\varepsilon TM^2}{2\varepsilon\varrho\phi M^2S(r+\varphi)}a_2+F,$$

$$\hat{g}=\frac{\delta M}{2\varepsilon\varrho M(r+\xi)}a_1+\left[\frac{\theta S}{2\phi\beta(r+\varphi)T}+\frac{(T+MS)\theta\eta\phi+\theta\varepsilon TM^2}{2\varepsilon\varrho\phi M^2S(r+\varphi)}\right]a_2+\frac{L}{\phi\beta T}+F.$$

Hence, Proposition 5 is proven.

G. Proof of Proposition 6 and 7.

Proof of Proposition 6 and 7 is similar to those for Propositions 4.

G. Proof of Proposition 8 and 9.

Proof of Proposition 8 and 9 is similar to those for Propositions 1 and 4.