A generative multilingual shape grammar for mini and micro-housing design and its robotic construction

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Sponsored by the British Council, this study focuses on a novel generative system based on shape grammar for housing. Shape grammar has been used over the last decades with the purpose of analyzing and recreating existing design languages. It has been documented the limited number of grammar used in new designs, implemented in computer tools and built. This paper reports the experience of grammar as a generative process, its implementation and a methodology to apply it using robotic construction. The main contribution is the construction of a prototype of a micro-house as generated by the grammar as the first real implementation of a grammar whilst using robotic concrete extrusion. The grammar as a generative system, allows for customization of housing whilst keeping affordability. Its computer implementation allows experts and laymen to design exploration. The grammar generated is also multilingual grammar allowing for the design of compact and micro-houses.

Design Computing and Cognition DCC'24. J.S. Gero (ed), pp. xx-yy.

Background

In design and computation, there have been many attempts to streamline housing. Automation and the use of robotics seem to be an efficient way to reduce construction costs, increase built quality, enlarge diversity, allow for customization and personalization of housing units.

This paper does not attempt to list all the efforts in the automation and customization of mass housing but will try to summarize relevant precedents.

The post-war housing effort that most of Europe was forced, either to rebuild or to face population growth experienced with the baby boom, allowed for many achievements in optimization.

In the Netherlands, post-war prefab repetition was the main strategy to optimize resources and reduce costs. The result was monotonous and not personalized making users feel annihilated and not comfortable in their environment. Habraken used that as a springboard to generate an expert design system based on a directing grid to allow for customization without compromising built agility and costs. In his work Support Systems, he describes a comprehensive system of grids that extends from structure to cladding, fixed furniture and internal partitions. [1]

Many were the rule-based systems implemented into housing. Expert systems to optimize mass housing are the most mentioned in research. Shape grammars are an efficient way of recreating such systems or even post-rationalize them.

Shape grammars are generative systems that use the notion of linguistics grammars applying it to design [2] [3]. In design, they assist in recreating a design family and/or a design language. This is translated into a set of grammar rules that use a lexicon of shapes and a syntax of geometric transformations. [4], [5].

Many were the grammars applied to housing, some describing the Palladian villas system [6] based on Palladio's design system expressed in his architectural treaty [7], the Frank Lloyd Prairie houses [8] or the first computer implementation of a three-dimensional shape grammar the Malagueira houses [9]. The above-mentioned housing grammars used the corpus of existing designs to post-rationalize the design system, infer design rules and recreate not only a set of existing designs but also a new corpus of novel solutions consistent with the original language.

Very little contribution has been made over the last decades in novel generative systems with grammars at their base. Often, grammar studies focus on analysis rather than on creating new generative systems able to create a new language to be applied in architecture. This is patent in recent surveys over shape grammar research which identified that over the last five decades since inception, grammarians have been focusing on either: analysis, research, case studies, and or education. The survey also cites that very little has been published regarding novel designs, particularly in the architectural field. [10].

Other interesting research has been produced regarding the optimization and use of digital tools for either the design or efficient construction of housing. Robotic construction and other automated forms of prefabrication have been developing new methodologies of design and assembly.

Prefab in housing construction is not new and is more or less frequent depending on its geographical location, North America probably leading the way in these efforts since the beginning of the twentieth century with the famous Sears catalogue houses that were flat-packed, shipped across the country and assembled in different states. Some of these houses have been commercialized since 1908 and have survived over 100 years. [11]

The flat-pack theme has become less popular over the last half of the twentieth century, whilst during the war Sears ceased manufacturing. The flat pack was far less popular in Europe where traditional construction methods and some highly mass-produced prefab remained the most common building methods. The flat-pack was however re-introduced recently with the Muji house experiment commercialized by the home goods store and the Wiki house. The latest is a zero-carbon, modular, open-source design system that enables self-build. [12] It proposes a flexible and customizable design system which allows competitive costs for affordable housing (approximately 185$ per sqft). All pieces are CNC machined and delivered ready to be easily assembled, allowing the choice of finishes and foundations. Major limitations are size, number of storeys (limited to 3 storeys), availability of main material (Orientated Strand Board – OSB or Spruce plyboard) and foundation types.

Other incursions using a proprietary commercialized system include a rule-based system with a computer tool that allows real-time modelling embedding the Actar and Manuel Gauza 'ABC' housing system. This work took an existing constructive system and integrated it with an architectural design system The tool allowed users the mass-customization of housing without additional costs. The real-time modelling allowed the planning of each housing unit while the overall building system and services were planned in real-time. The output was a three-dimensional detailed model with structure, infill, finishes fixed furniture/appliances, technical drawings including plans, sections and elevations and a list of components with their key dimensions ready for automated production, a study that was a predecessor to mainstream BIM tools. [13]. The building would then be assembled as a kit of parts on-site,

More recent experiments are testing new forms of utilizing robotic means. Three-dimensional printing became popular in the academic field and within forefront practices that are leading material research. One of the most popular methods is concrete extrusion using robots equipped with concrete pumps and nozzles. The benefits of this additive construction method are fourfold:

1. Expedite construction process
2. Facility of construction complex geometries
3. Optimization of the use of material
4. Reduction of material waste usually used for formwork and reduction of concrete and steel reinforcement overall reducing the embodied carbon associated with construction.

Some of the more interesting solutions have been the on-site construction using robotics such as the Icon gantry-operated concrete nozzle. Using this method several houses have been extruded and built in the US with great architectural and built quality. The speed of the process in comparison to traditional construction is impressive with a single-story house built within a few months. This also allows for an expedited process from conception to design development and construction/use. [14].

Most of these experiments rely on concrete extrusion with a strategy based on the sheer wall loadbearing properties of concrete in compression but interesting experiments have been made using mixed strategies where reinforcement is included in the fabrication process such as experiments at ETH Zurich. [15] This experience relied mostly on off-site construction and on-site assemblage using a mix of strategies.

Similar strategies in robotic construction included a cabin in the woods by research and practice Hanaa in Ithaca. Using Cornell's robotic lab, the architects proposed rapid prototyping of 3D extruded concrete in panels and printed elements later assembled on-site to create a small micro-home or studio space for work and contemplation. The proposal was finished with timber planks rustically cut from deformed tree trunks and applied as cladding. The work did not result from a rule-based system nor used any generative process to be conceived but used digital tools for optimization, modelling and planning for automated construction.[16]

Given the use of reinforcement and the need for transport, the embodied carbon of some of these solutions requires attention.

Other more radical experiments are planning for on-site robotic construction in remote construction, even trans-planetarian experiments where scarcity of materials, means and capacity of transport might be an issue.

The London-based firm Foster and Partners in collaboration with the European Space Agency proposed a Lunar Habitat that used the mentioned restrictions as a design concept. A very compact shuttle allowed for the transportation of materials and machinery to the Lunar surface containing a 3D printing extrusion nozzle and robot, and a mixture of regolith (lunar aggregate) and a concrete mix allowed for the creation of habitats. No formwork was required, instead an inflatable membrane could be propped and once in maximum expansion would serve as the base for the 'concrete' deposition process. Once finished the entrance portals transported to the site would remain and the 'shuttering' membrane the internal finish. From the outside, the habitat would blend with the surrounding landscape. [17] [18].

Other similar futuristic experiments were also proposed, this time sponsored as part of a NASA competition for Martian habitats. In this case, several academic and commercial teams took part but one of the finalist teams proposed an innovative solution using extruded concrete. A team led by Penn State University proposed a solution where each design module had a self-supporting conic structure with a gradient material that ranged from opaque to transparent, able to provide thermal insulation and shelter against radiation. The solution once more proposed the use of local aggregate mixed with concrete and extruded. The geometry of each unit was carefully tested so that the apex of the cone allowed for each layer of extruded concrete to support the layer above whilst still wet and flexible, while still providing a viable structure once cured. Although far from being executed on-site, this proposal had a series of test run examples at a 1:3 scale where prototypes were successfully erected by robotic arms in a controlled environment. [19], [20]

What both experiments have in common is the use of robotic construction in an inhospitable environment where human life would be difficult and labor-scarce. What we can learn from both experiments is how to use robotic construction when and where construction can be optimized, rationalized, expedited and made more affordable.

For this reason and following a call out from the British Council a team of academic researchers was inspired to propose the use of robotic construction to streamline affordable, mass-customized sustainable housing. The aims of the study are cited in the section below.

Aims

The great housing problem that affects big urban centers is complex and cannot be tackled with current already-tested solutions. The construction process is long and expensive and the process from conception to planning is unattainable to most. For these reasons is necessary to think outside the box and deliver new holistic approaches that revolutionize the market from inception to conception, from design to construction, trimming all entropic stages and unnecessary stages.
An innovative approach will include the following objectives:

- Create a new design grammar for generative system purposes
- Use a new grammar for design exploration
- Grammar implementation using a design script
- Build a real-scale prototype using a design generated from a grammar
- Use novel methods of robotic construction – 3D printing

Many are the shape grammars that cover housing design, such as the Taiwanese houses [21], the Queen Anne houses [22], and the Buffalo bungalows [23] to name a few, but what all of these grammars have in common is the attempt to recreate a design language tested and built. Very few examples focused on architectural grammars as generative systems despite its proven record of potentiating design and allowing design exploration. Grammars are a valid means of achieving mass-customization of housing whilst maintaining a consistent language and a valid relationship of spatial adjacencies as long as the rule inference is suitably arranged.
The contribution of many researchers to grammar implementation has been notorious. We highlight the role that Gips had in grammar implementation using computer tools [24] during a computing workshop at MIT, which inspired the Malagueira implementation of the first housing shape grammar to be implemented in a real-time three-dimensional modelling tool. [9] followed by many other implementations such as Jowers applied to Islamic tile patterns, Stouffs and Economou grammar implementations. [25], [26], [27], [28].
The implementation of grammar in computer tools to this point is either motivated by a specific design language or abstract design tools that allow design exploration and shape embedding. Despite the success of all these tools, the current study proposes a specific tool that implements the proposed system and potentiates laymen the perusal and application of the generative system using a common modelling tool Rhino and using Rhino Script for its coding.

Another aim of the current research project is the real-life application of both grammar as a generative design system, computer implementation to enable exploration and design diversification and its construction. Aided by British Council funds and academic funding a real-scale prototype is aimed to be built within the University of East London Campus using the robotic fabrication lab DFUEL and the equipment available an ABB robot, concrete extrusion nozzle and concrete pump. The construction of the real-scale micro-house will take place in the summer of 2024. This is to the extent of our knowledge the first application of a shape grammar for design purposes that is erected, and in this case, combines robotic construction.

The role of robotic construction is fourfold it aims for cost reduction, waste reduction, embodied carbon reduction, and construction efficiency even with complex geometries.

Method

The combination of a generative shape grammar as a novel design system and robotic construction encompasses a working methodology that is threefold:

1. Shape grammar as a generative system
2. Implementation of the grammar into a design script
3. Creation of a chosen design and construction

1. The generative system uses a three-dimensional grammar system to generate novel designs. Due to the complexity of the project and involved costs, it was important that the grammar was multilingual allowing for the design of both compact affordable houses but also micro-houses. The purpose of a micro-house was solely to facilitate its design and construction as a proof of concept. The shape grammar is illustrated in Figure 1 where a selection of the additive grammar rules is illustrated. The grammar approach is bottom-up and uses a three-dimensional representation. The houses are modular, growing progressively by placing different functional modules in adjacency. The design is progressively made from social areas to services (kitchen and bathrooms) and ends in the most private bedroom areas. The size of the main portal frame for each module varies from a modest 2.40m (the width of a shipping container) to 4.20 m the absolute maximum width to be transported by land on the back of a lorry, if prefab is the strategy of choice rather than on-site robotic construction.

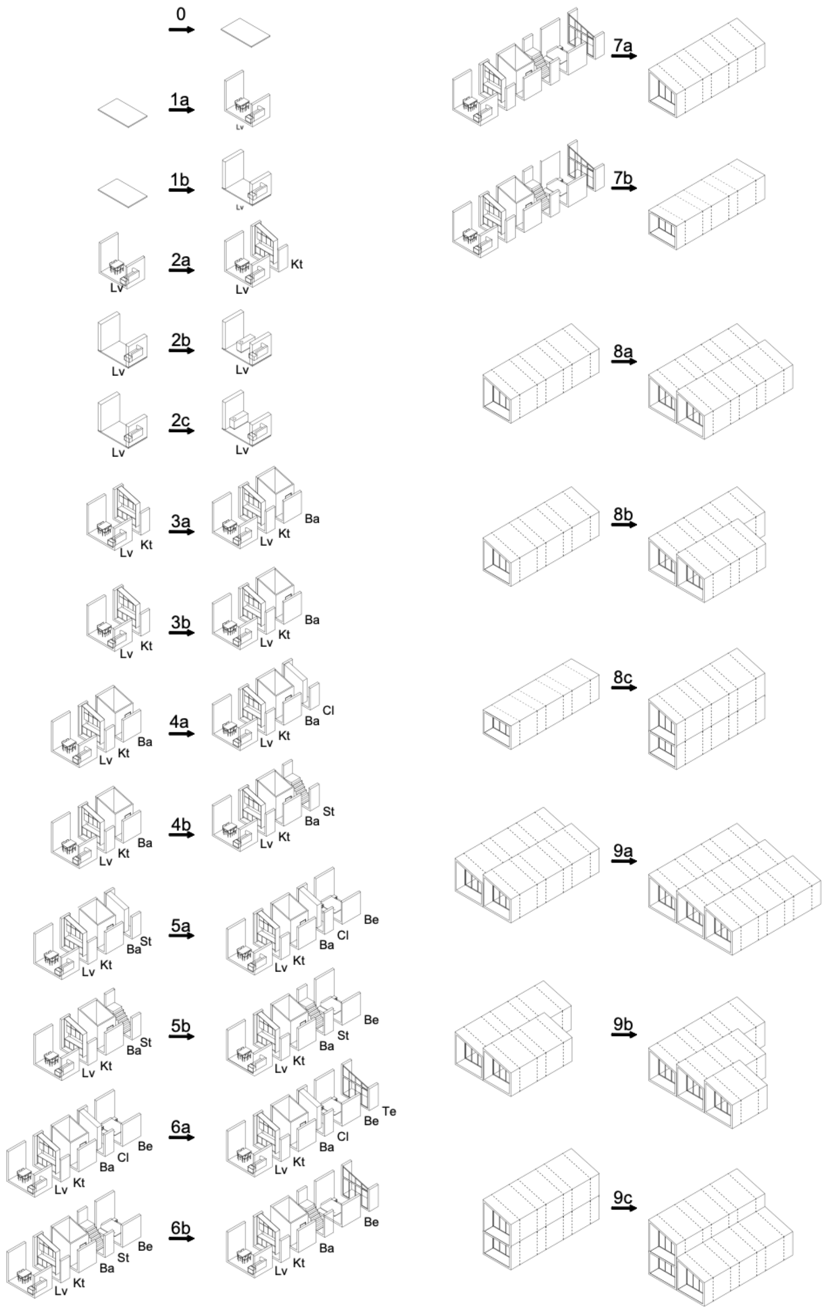

Fig. 1 Multilingual shape grammar rules for mini and micro-housing, partial representation of the rule system

The tri-dimensional grammar representation allows for an easy understanding of the system composed of 9 design stages. The length of the module varies from 1.2 m for a basic kitchenette or stair unit to a more generous modular multiple of 1.2 such as a minimum bedroom of 2.40 m.

Once one or more modular units are assembled, they allow for a housing unit in a tubular disposition. One to three of these modular arrangements are allowed as observed in the rules. The rules are parametric and allow the modularity discussed compatible with most building materials.

Multilingual grammars are grammars that enable the design of multiple trends while keeping the design consistency. While original grammar like the Palladian grammar allowed for the design of Palladian villas i.e, a multilingual grammar like the proposed, allows for the design of modular housing with a specific expression but aggregates three different typologies addressed in parallel grammars, normal, compact and mini-housing. These branch out in different ramifications as the tree diagram illustrates in Fig 2.

The rules are numbered according to the stage of design. Besides their parametric feature, they manage the configuration of spaces and the functional layout of each housing unit. While rules 0 to 7 are responsible for interior spaces, rules 8 to 9 a, b, and c relate to enclosures of roofs and envelopes. These rules are also responsible for the scale-up of the housing units by the adjacent placement or pilling up of units. The grammar allows for several designs both with sharp edges and curved connections targeting both conventional prefab assemblies in metallic structure or cross-laminated timber assemblies or robotic construction with concrete extrusion typically related to curved detailing. The grammar caters to design flexibility but plans for expedited means of construction.

Although robotic construction costing data is to this day difficult to ascertain, there are easy wins that can be predicted. Robotic construction allows for reduced labor costs, reduced construction time, reduced construction wastage by efficient planning or usage of computer means, reduction of carbon emissions by reducing the amount of concrete when compared with conventional reinforced concrete, reduction of rebar and elimination of formwork, shuttering and/or hoarding costs. If required prefabrication transportation will have to be considered.

The proposed grammar is deterministic and proposes a finite design corpus, being not as prolific as other grammars, however, it supports a generative system for novel housing to be erected which is believed to not have been attempted before.

2. The grammar implementation was carried out in Rhino, using RhinoScript in VBA. The purpose is to use the modelling opportunities generated by Rhino and be able to control the outcomes and any potential

change allowed by the parametric design system. This could also be made using Grasshopper and or RhinoPython, but due to the geometries used and the challenges faced the most suitable solution.

The script follows the structure of the grammar as illustrated in Figure 2. In this tree diagram, a series of processes or stages are described:

1. Stage 1: First unit width and length definition (min 2.40 x 1.20 m)
2. Stage 2: First functional unit definition – Living area
3. Stage 3: Definition of second unit – Living area extension or kitchen
4. Stage 4: Definition of bathroom unit
5. Stage 5: Definition of closet or staircase area
6. Stage 6: Placement of bedroom and additional extension if required
7. Stage 7: Placement of end terrace and bedroom glazing
8. Stage 8: Conclusion of modular unit and roof encasing
9. Stage 9: Design of basic house extension, roof and envelope or enclosure (Stages 1 to 8 recursively)

The grammar structure illustrated showcases the multilingual aspect of this grammar. Using similar rules one can potentiate the design of mini or micro-homes. A smaller home can be attained from a reduction of the parametric values to the minimum allowed by the system. However, this could imply and correlate to a reduction of useful internal space or a reduction in spatial quality. It was important that the micro-home also maintained a minimum of spatial quality and aesthetic. Therefore, the grammar branch illustrated on the left refers mostly to larger typologies whilst the right wing to the micro-homes. Although the same shape rules can be used for both house sizes and typologies a more expedited grammar derivation allows the design of micro-homes. This can be observed in detail in the tree diagram that illustrates the micro-homes sub-grammar in Figure

This shows the possible process of designing the smallest 'home' or studio space. The four cases illustrated do not propose a bathroom but are the absolute smallest modules and the four options are equated for proof of concept and construction. More generous configurations are available for 'operational' micro-homes that showcase sleeping, living, bathroom and kitchen additional modules designed by specific rules are available.

Figure 4 explains the graphic process of derivation of one of these micro-homes in six steps. By applying a simple module, façade and roof structure possible in the system this compact configuration is attained.

The two steps in this process relate to planned tasks that are yet to take place in the following months.

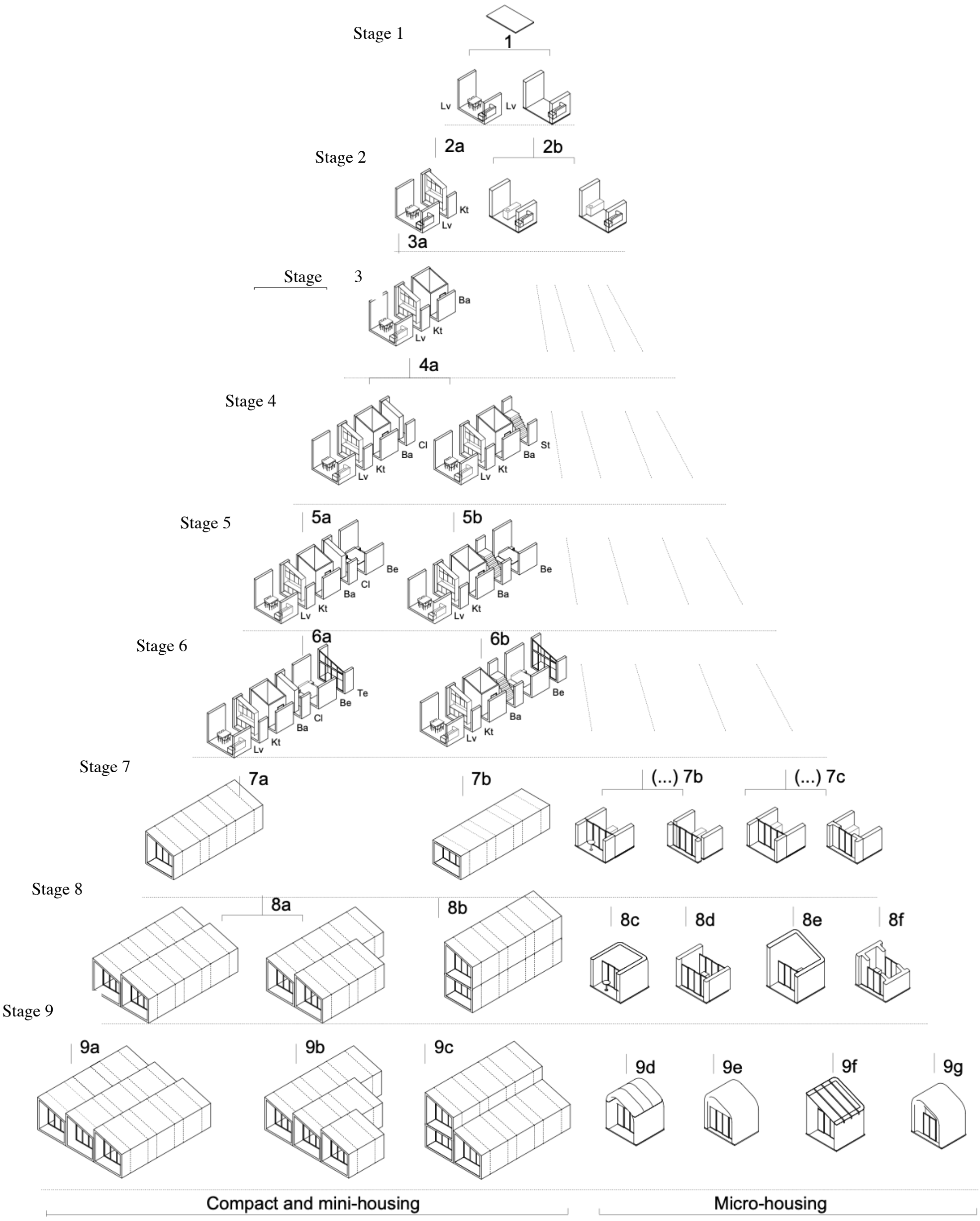

Fig. 2 Multilingual grammar tree structure for the design of mini and micro houses

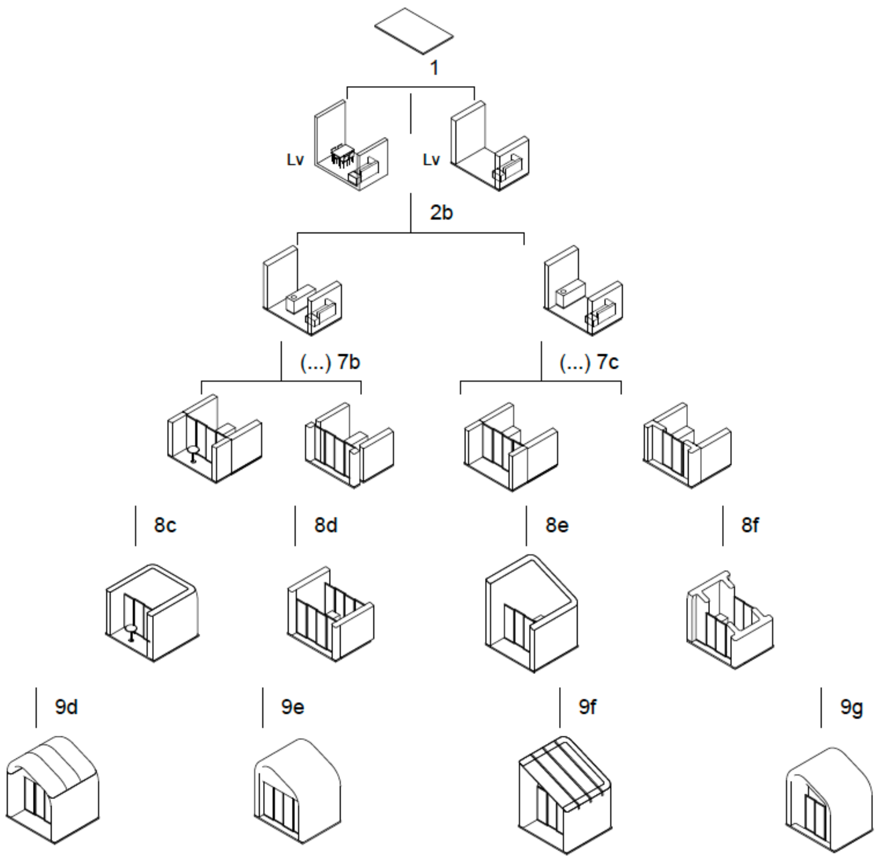

Fig. 3 Micro-homes sub-grammar structure

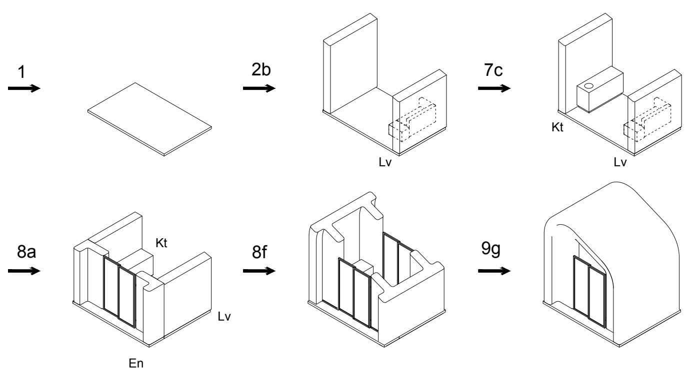

Fig. 4 Micro-homes sub-grammar shape rules within a derivation of a micro-house

This includes:

1. Printing of the prototype in scale
2. Construction of the real-scale house on site

Both tasks will take place between January and September 2024 in the academic environment. The prototype printing of the mini-house already occurred, but a more detailed model of the micro-house is yet to occur. The shortlist for the development of the micro-house is shown in Figure 5. This sample shows four designs attained by the grammar and modelled three-dimensionally. Bellow in Figure 6 the corresponding floorplans are shown. They show similar compact units of 8 sqm each and some with enlarged external areas such as front porches. All options are illustrated with a monolithic construction where the external wall is printed using robotic construction with additive methods of concrete extrusion.

The erection of the real-scale micro-house will take place in June during an international workshop in Shanghai with Tongji University, where 20 students from two universities will participate supported by the technical staff of the robotic fabrication lab. In this workshop, the micro-house will be constructed and exhibited as part of the London Festival of Architecture throughout the initiative.

This aims to be the first build solution resulting from a generative system such as a shape grammar allied to robotic construction.

On the other hand, the system also allows for larger solutions. For larger solutions, robotic construction on-site or off-site are both available with the limitations associated with transportation. Figure 7 shows the derivation process by rule application to generate a 96 sqm house unit. A working bay of that same unit is then shown in Figure 8 where the spatial articulation as resulted of the generative grammar is also shown. This shows the flexibility of the system and its potential for future expansion.

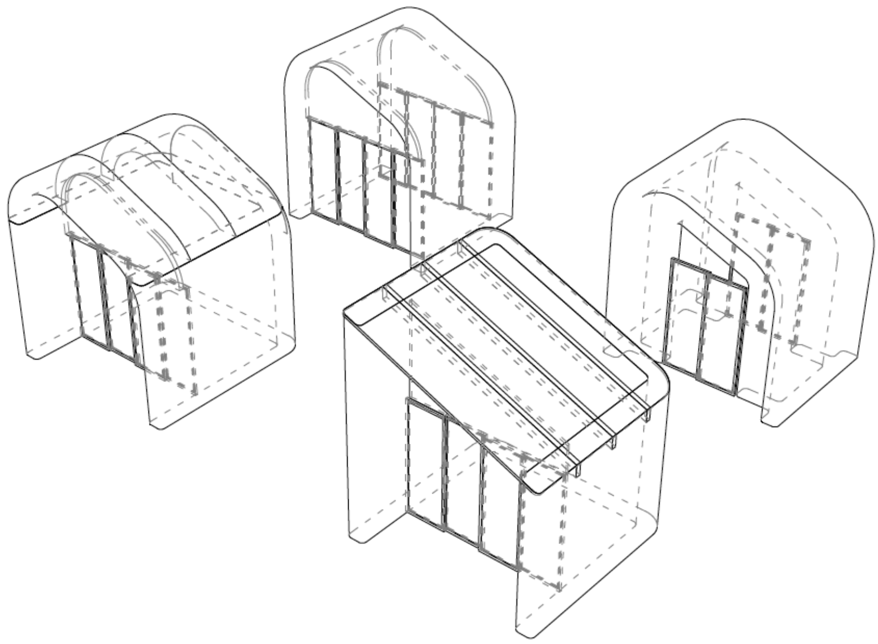

Fig. 5 Four design solutions to illustrate the micro-homes grammar

Fig. 6 Four design solutions of micro-homes grammar and their floorplans

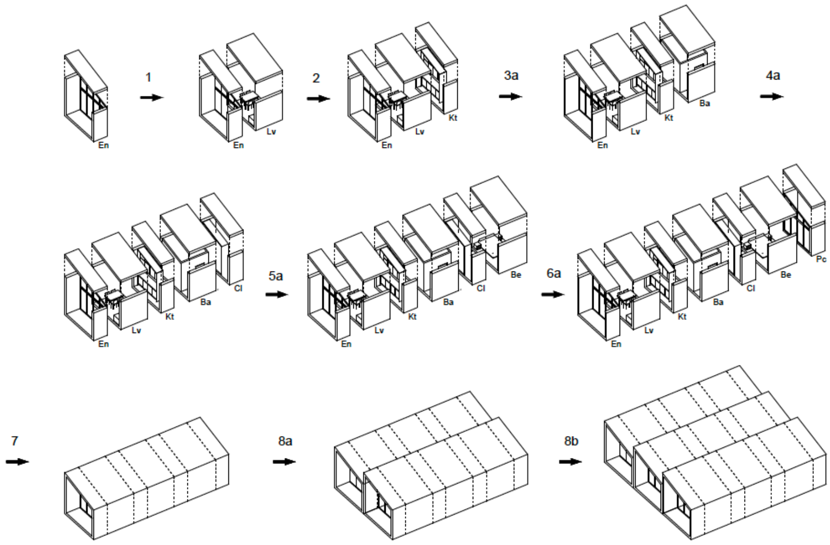

Fig. 7 Housing unit for a mini-home step-by-step derivation

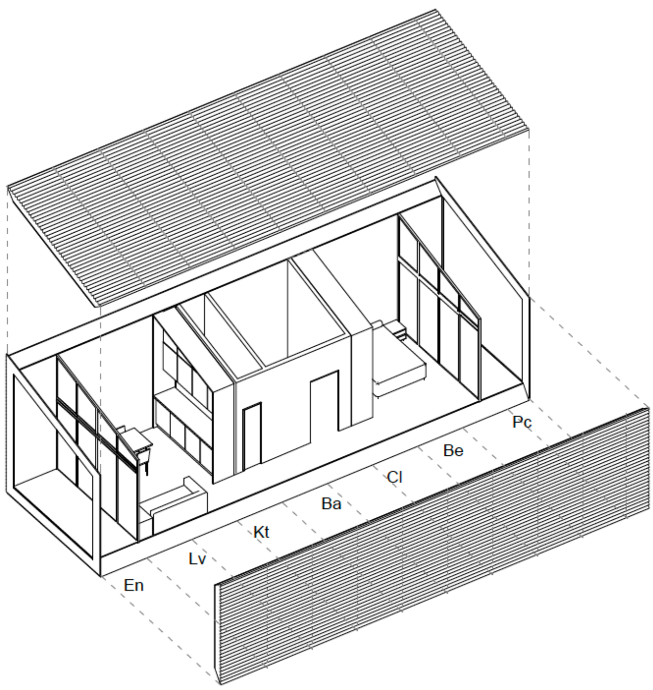

Fig. 8 Mini-homes exploded axo with internal spaces (approximately 32 sqm)

Results

This paper focuses mainly on steps 1 and 2 of the wider research which will address robotic construction. Figures 9 and 10 showcase 4 physical models of design solutions 3D printed in the digital lab at 1:100 scale. The physical prototyping of the micro-home will take place in August 2024 although concrete extrusion testing has started as Figure 11 illustrates. Currently, the viscosity, correct mix ratio and speed of production are being tested. For this reason, the current paper discusses the generative system and the computer implementation.

The use of grammar as a generative system has been limited, while most focus on analytical or recreation tools. Even more rare are real-life examples of buildings erected using grammars as generative systems. This multilingual grammar targeting both mini and micro homes has shown the required flexibility to produce a wide range of highly customizable solutions.

Despite the constructive process selected, the grammar itself would be suitable to other structural systems, typical offsite and even prefabricated volumetric strategies. Its design and system allow a high level of flexibility where the designer/user can design consistent options and select a suitable constructive system that is relevant or appropriate to the geometric locations, cost-effective and available.

Although we cannot comment on step 3 – robotic construction, this prototyping experience on a real scale will shed light on the feasibility of the proposal and its real limitations. Concrete extrusion testing has started in preparation to produce a real-scale prototype. The results of the future work are still pending but will certainly illustrate the full process of a fully automated design system from conception to construction.

Many are the unknowns and potential risks associated with the first attempt to build with robotic construction and it will certainly lead to interesting discussion for future work.

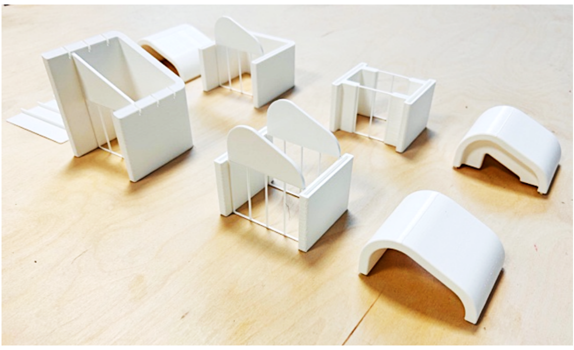

Fig. 9 Micro-homes exploded 3D printed physical models

Fig. 10 Micro-homes 3D printed physical models

Fig. 11 Digital lab - robotic arm ABB and extrusion nozzle in testing stage

Conclusion

This paper describes the design methodology of a real-life grammar to be applied not to analytical post-rationalization of design but to be used as a generative design system. This methodology becomes even more powerful when taking full advantage of digital tools, grammar implementation and automated construction using robotic means. The contribution of this paper is threefold:

1. Creation and implementation of a grammar-based generative system
2. Customization of mass-housing
3. A streamlined methodology using robotic construction.

This proof of concept has proved to be an efficient way of streamlining the design process while attaining a quality, consistent design, that caters to the needs of the occupants and allows for mass customization. The grammar was implemented on a computer tool running in the mainstream Rhino allowing for the real-time modelling of the generated solution. This can be evaluated by the user and assessed for its suitability. Other implementations were proposed in the past but very few ones allow the creation of new designs. The project also caters for the customization of housing units.

A current problem of housing design is that both ends of the housing spectrum are normally addressed. Highly specified and personalized designed houses are expensive and not widely available while to this day affordable housing is linked with poor design quality, standardization, repetition, and lack of customization. This proposal aimed at catering for the need for high quality, suitability and personalization. This was achieved by a modular design of elements that can be personalized, based on a generative model supported by a parametric grammar implementation. This allows each user to consciously select the configuration that is more suitable while controlling the maximum area of construction. Each house can then be expanded over time with the addition of additional bays. Finally, the use of robotic construction which is currently being timidly introduced in commercial experiments is the end goal.

The use of robotic means also results in a streamlining of resources, reduction of construction material wastegate and reduction of construction time.

The first robotically produced prototype will be built in an academic environment at a 1:1 scale during the summer of 2024 to showcase the process.

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