Complexity Reduction in Beampattern Matching Design for MIMO Radars

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***Abstract*—Covariance matrix design and beamforming in multiple-input multiple-output (MIMO) radar systems have always been a time-consuming task with a substantial number of unknown variables in the optimization problem to be solved. Based on the radar and target conditions, beamforming can be a dynamic process and in real-time scenarios, it is critical to have a fast beamforming. In this paper, we propose a beampattern matching design technique that is much faster compared to the well-known traditional SQP (semidefinite quadratic programming) counterpart. We show how to calculate the covariance matrix of the probing transmitted signal to obtain the MIMO radar desired beampattern, using a facilitator library. While the proposed technique inherently satisfies the required practical constraints in covariance matrix design, it significantly reduces the number of unknown variables used in the MSE (minimum square error) optimization problem, and therefore reduces the computational complexity considerably. Simulation results shows the superiority of the proposed technique in terms of complexity and speed, compared with existing methods. This superiority increases by increasing the number of antennas.**

***Index Terms*—Covariance matrix design, real-time beamforming, multiple-input multiple-output (MIMO) radar, SQP (semidefinite quadratic programming), MSE (minimum square error).**

I. Introduction

MULTIPLE-input multiple-output (MIMO) radar systems have been attracting considerable attention recently due to their performance advantages such as higher spatial resolution, improved target identifiability, waveform diversity and flexibility to design a variety of transmit beampatterns [1]–[3]. They have the potential to significantly improve radar remote sensing performance in a number of important applications including airborne surface surveillance, over-the-horizon radars [4]–[6] and tracking multiple targets [7]. The concept of MIMO systems is not new. MIMO techniques have experienced great success in other radio frequency (RF) systems, mostly in wireless communications. The important enabler features for both radar and communication systems to benefit from MIMO techniques are generally the same, however, the performance metrics and implementation approaches are quite different. In communications systems MIMO antennas enable improved channel capacity in complex propagation and scattering environments dominated by multipath propagation. Similar to MIMO communications that can develop wireless network and improve capability of communication, MIMO radar systems can achieve high performance in signal processing [8], [9]. MIMO radars have numerous advantages over uniform phased array (UPA) radar counterpart; namely decreasing sidelobe levels (SLLs) in designing beampatterns, reducing the signal-to-interference-plus-noise ratio (SINR) and controlling cross-correlations. Radars with UPA antennas transmit fully correlated waveforms with possibly different phases and amplitudes, therefore they are considered to be single-input single output (SISO) systems. In MIMO radars the waveforms that are transmitted by different antennas are orthogonal or have a percentage of orthogonality, and this degree of freedom provides MIMO radars some features that do not exist in SISO systems, the most important of which are vast field of view and virtual arrays [10], [11]. MIMO radars can be classified into two types: widely distributed and collocated MIMO radars. In widely distributed antennas, array elements are physically separated with a large space, enough for each antenna to see different targets' radar cross-sections (RCSs). These radars can enhance spatial diversity as well as detection. In collocated MIMO radars the transmitting antennas are located at small distances from each other, and their transmitted waveforms can be completely independent (or orthogonal), partially correlated or completely correlated. There are other categories of MMO radar systems such as monostatic, bistatic and multi-static. Monostatic MIMO radars use unique antennas as transmitters and receivers. In bi-static radars, the transmitters are in one place and the receivers are in another place, and multi-static radars are those that have several transmitters in one place and several receivers in another place. In this paper, the focus is on collocated and monostatic MIMO radars.

In MIMO radars, waveform design plays a significant role in achieving desired advantages in numerous applications such as beamforming with integrated sidelobe levels (ISLs) constraints [12], antenna selection [13], spectrally compatible applications [14], [15] and etc. The collocated MIMO radar waveform design can be divided into two categories; one on the receiver side for achieving maximum output SINR [11], [16], [17] which can enhance the target detection performance and suppress signal-dependent interferences. The other is to design MIMO radar waveforms with desirable transmit beampattern to control the radiation power distribution on the transmitter side. The second category itself can be classified into two subclasses of direct and indirect waveform design. In direct

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from antenna to obtain desired characteristics. Therefore, the main aim in optimization problem is to design the probing waveform directly, however, indirect waveform design refers to design of the covariance matrix in the first step, then deriving the waveforms from the achieved covariance matrix in the second step. Therefore, the waveform covariance matrix design problem is considered as the main aim by proposing the transmit beampattern design metrics to match a desired transmit beampattern or to solve the minimum square error (MSE) problem and minimize a penalty term via optimization methods. The penalty term is the cross correlation of composing different direction signals for defined angles in the region of interest. In direct waveform design problems, constant modulus (CM) and positive semidefinite (PSD) constraints are the practical constraints that mostly considered in waveform design optimization problem. CM is a practical constraint for increasing the efficiency of power amplifiers (PAs) used behind the antennas in MIMO radars [18]. It applies equivalent waveform transmission from antennas elements in each time (or sample). In indirect waveform design problems, PSD constraint is essential for the realization of waveforms and the constraint of equality of the diagonal elements of the covariance matrix which is necessary for avoiding the destructive effects of the PAs operation in the saturation region. For instance in the direct waveform design problem, [19] considered waveform design in space domain by selecting appropriate phases of the waveforms from all MIMO radar antenna elements to obtain the desired beampattern. In their proposed algorithm, the waveform transmitted from an antenna element must be coded in time domain to be orthogonal or nearly orthogonal. In [20] the binary transmit waveforms are designed via minimizing the beampattern integrated sidelobe to mainlobe ratio and likelihood ascent search (LAS) method is used as a solution for optimization problem. Additionally, [21] applies two algorithms based on the alternating direction method of multipliers (ADMM) method to obtain the desired beampattern in directly probing waveform design. The ADMM algorithm is a distributed optimization approach with the numerical robustness of the augmented Lagrangian method. Based on the ADMM framework, nonconvex polynomial functions and nonconvex multi-constraint problems could be solved [22].

Indirect waveform design is considered in the literature such as [23] where the covariance matrix design is stated as a semidefinite quadratic programming (SQP) problem for obtaining the elements of a square-root matrix of the covariance matrix based on parametrizing coordinate of a hypersphere with considering the practical constraints. In [24] Toeplitz matrices are proposed for MSE problem with low computational complexity while fulfilling the necessary constraint. Also in [25] the iterative methods are utilised for designing the covariance matrix and used the barrier method as a simple technique for solving convex optimization and synthesise BPSK waveforms which satisfy CM constraint. [26] is used to convert a constraint problem to an unconstraint one and extract the covariance matrix. In [27], [28] a closed-form method is presented to design the covariance matrix for a uniform linear array that uses the discrete Fourier transform (DFT) by reducing the complexity of the iterative methods. Also in [29], [30] several covariance matrixes are proposed that satisfy PSD and the equality of the diagonal elements of the covariance matrix constraints and then the BPSK waveforms which realise these covariance matrices are generated. To this end, synthesising transmit waveform under practical constraint (CM or peak to average power ratio (PAPR)) after obtaining covariance matrix is reviewed in [25] and [30], [31]. [32] applies cyclic algorithm (CA) to generate CM waveform with specific covariance matrix. In [33] an effective algorithm is proposed based on multi variable optimization problem of designing the CM transmitter waveform for a collocated MIMO radar to meet a desired output beampattern with both indirect and direct waveform design methods. Waveform covariance matrix design problem in different scenarios is solved in [34] by well-known SQP method, where different covariance matrixes are generated in different applications. Two important form of objects that are solved by CVX [35] and considered in [34] are beampattern matching design and minimum sidelobe level problem. However, in [23], [34]–[36] the focus is on minimising the mean squared error between the achieved beampattern and the desired beampattern, and little attention is paid to SLL suppression, mainlobe ripple constraints and wide beampattern design.

In this paper, we focus on the indirect waveform design in collocated and monostatic MIMO radar. We propose a new simple and fast approach for solving MSE problem to design the covariance matrix under practical constraints. The main contribution of this paper is to provide a fast and real-time covariance matrix design method to achieve the desired beampattern with appropriate specification in the mainlobe and sidelobe levels, using our proposed facilitator libraries. We introduce the facilitator libraries that include a set of the covariance matrixes which are designed based on simple UPA structure as UPA library and minimum sidelobe level beampattern design problem solution [34] as SLL library. In the proposed technique, the desired covariance matrix in MSE problem can be written as a linear combination of stored covariance matrixes in the library with corresponding coefficients. Then we propose an algorithm to find the coefficients as unknown variables by solving an optimisation problem to achieve the best covariance matrix in beampattern matching design problem. In this approach, we will show that we can achieve a significant reduction in the number of unknown variables, a considerable decrease in the computational complexity and a great reduction in the time consumption compared with well-known counterpart SQP, and when the number of antennas is high, this difference is more considerable. The novel aspects of the proposed technique can be highlight as follows:

- Beampattern design in radar systems should be fast and therefore, providing a method for fast and real-time waveform design is one of the real-time system requirements. The proposed technique in this paper reduces the number of variables in the optimization problem, and hence reduces the complexity and increases the speed of algorithm compared with other methods in the literature.

- PSD constraint and the uniform elemental transmit power constraint for proposed covariance matrix are already satisfied for all matrixes stored in the proposed facilitator library, and these conditions also exist for the linear combinations of the called covariance matrixes of the libraries. Therefore, by establishing these two practical conditions in the library we can omit them in the optimisation problem, and this makes much easier to solve the corresponding optimization problem.

The rest of the paper is arranged as follows. The problem formulation is presented in Section II. The proposed technique in optimal design is explained in Sections III. Simulation results are illustrated in Section IV and finally conclusions are drawn in Section V.

*N*otation: Bold upper case letters $\mathbf{X}$ and lower case letters x, respectively denote matrices and vectors. Conjugate transposition, conjugate and transposition of a matrix denoted by $(.)^H$, $(.)^*$ and $(.)^T$, respectively, and statistical expectation is denoted by E$\{.\}$. The *(m,n)th* element of a matrix is denoted by $\mathbf{X}_{mn}$. $\mathbf{X} \geq 0$ shows the matrix $\mathbf{X}$ is PSD.

II. PROBLEM FORMULATION

Consider a collocated and monostatic MIMO radar system with M number of transmit antennas that are placed in distance d (it could be assumed half wavelength). Let be the discrete time radar waveform radiated of mth antenna for m = 1, …, M. Also, n = 1, …, N denotes the number of samples of each radar waveform transmitted by each antenna. It is assumed that the transmitted signals are narrow-band and the propagation is non-dispersive. The steering vector of the array is defined as follows:

$$a(\theta) = [1 \quad e^{j2\pi \frac{d}{\lambda}\sin(\theta)} \quad \dots \quad e^{j2\pi \frac{(M-1)d}{\lambda}\sin(\theta)}]^T \tag{1}$$

where λ is the carrier signal wavelength and the baseband transmitted signal vector at each time for the nth sample can be written as:

$$x(n) = [x_1(n) \quad x_2(n) \quad \dots \quad x_M(n)\ ]^T \tag{2}$$

Under the assumption that the transmit antenna of the MIMO radar systems is calibrated, that is, $a(\theta)$ is a known function of θ, the received signal by a target located at angle θ can be given by:

$$r(n;\theta) = a^T(\theta)x(n) \tag{3}$$

Therefore, the transmit beampattern of the transmitted signal at the location θ is given by:

$$P(\theta) = E\{a^T(\theta)x(n)x^H(n)a^*(\theta)\} = a^H(\theta)\mathbf{R}a(\theta) \tag{4}$$

where $\mathbf{R}$ = E$\{x(n)x^{\mathrm{H}}(n)\}$ is the covariance matrix of the transmitted waveforms. In the indirect method, where the beampattern design problem is to synthesize covariance matrix, the equality of the diagonal elements of covariance matrix and PSD constraints as practical constraints can express respectively as:

$$\mathbf{R}_{mm} = \frac{c}{M}, \qquad m = 1, \dots, M \tag{5}$$

$$\mathbf{R} \geq 0$$

where $\mathbf{R}_{mm}$ denotes the *(m,m)*th element of $\mathbf{R}$ and c is the total transmit power of the array. This practical constraint is essential for efficiency of PAs in MIMO radars. Next the problem of finding $\mathbf{R}$ in indirect method for achieving desired beampattern can be expressed as two states: beampattern matching design and minimum sidelobe level beampattern design [34] which are described as follows.

A. Beampattern Matching design

In the beampattern matching design problem, the goal is maximising the total spatial power at a number of given target locations, or match achieved beampattern with a desired one as $P_d(\theta)$ which is known by MSE problem. We assume that a grid of target locations is over $\{\theta_k\}$ where $k = 1, \dots, K$, with the total number of given target locations K. The aim is to choose the best covariance matrix $\mathbf{R}$ such that the achieved beampattern by (4) matches to the desired beampattern $P_d(\theta)$ over the range of interests. Therefore, the main problem is considered as:

$$\min_{\mathbf{R}} \quad \frac{1}{K}\sum_{k=1}^{K}\omega_k[P_d(\theta_k) - a^H(\theta_k)\mathbf{R}a(\theta_k)]^2 \tag{6}$$

$$s.t \quad \begin{array}{l} \mathbf{R}_{mm} = \frac{c}{M}, \qquad m = 1, \dots, M \\ \mathbf{R} \geq 0 \end{array}$$

where $\omega_k \geq 0$ is the weight for the kth space grid point. If we assume that the covariance matrix for M antennas is symmetric, there would be $\frac{M^2-M}{2}$ complex unknown variables to solve (6) by CVX toolbox [35].

B. Minimum sidelobe level beampattern design

In some applications in MIMO radar systems, the beampattern design problem is considered as minimising the SLL in a certain angle range, when pointing the MIMO radars toward single angular (only like θ_0). Minimum SLL beampattern design problem, with PSD constraint and the uniform elemental transmit power constraint for covariance matrix, can be expressed as:

$$\begin{aligned} &\min_{t,\mathbf{R}} \quad -t \\ &subject \quad to \\ &\qquad a^H(\theta_0)\mathbf{R}a(\theta_0) - a^H(\mu_l)\mathbf{R}a(\mu_l) \geq t \quad \forall \mu_l \in \Omega \\ &\qquad a^H(\theta_1)\mathbf{R}a(\theta_1) = 0.5a^H(\theta_0)\mathbf{R}a(\theta_0) \\ &\qquad a^H(\theta_2)\mathbf{R}a(\theta_2) = 0.5a^H(\theta_0)\mathbf{R}a(\theta_0) \\ &\qquad \mathbf{R} \geq 0 \\ &\qquad \mathbf{R}_{mm} = \frac{c}{M} \quad m = 1, ..., M \end{aligned} \tag{7}$$

where $\theta_2 - \theta_1 (\theta_1 < \theta_0 < \theta_2)$ determines the 3 dB main bandwidth and is a discrete angle range that covers the sidelobe region of interest. While this problem is a semi-definite program (SDP) and can, therefore, be efficiently solved numerically, it does not seem that have a closed-form solution. Therefore, the goal here is to choose an $\mathbf{R}$ that makes minimum sidelobe beampattern in (4). This problem can be solved by CVX toolbox [35].

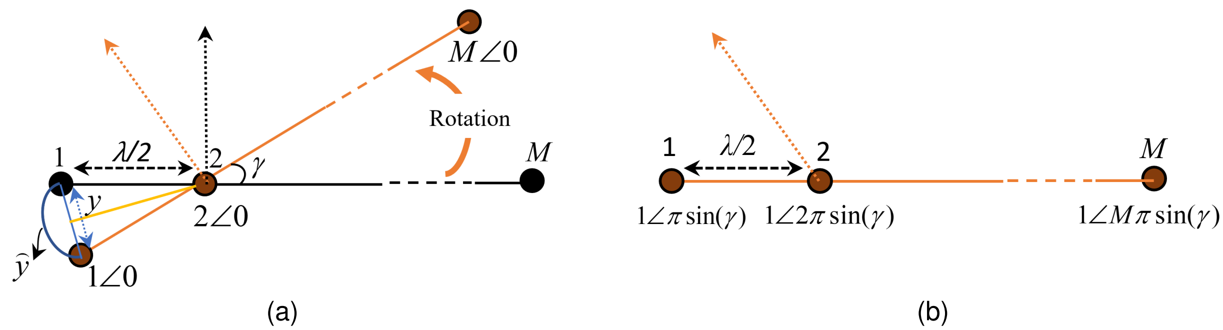


Fig. 1. (a) An example of UPA antennas and beampattern configuration and mechanical rotated systems, (b) electrical rotated UPA as a substitution.

III. PROPOSED METHOD IN OPTIMAL DESIGN

In this section, we describe our method for covariance matrix design using the proposed library. In a given beampattern matching problem with a desired beampattern $P_d(\theta)$, our focus would be to solve the MSE problem to obtain the covariance matrix in a simpler and faster method and get $P_d(\theta)$ using two facilitator libraries. Our proposed libraries include a set of certain covariance matrixes. These stored covariance matrixes satisfied practical constraints and can be created based on specific parameters like focusing on mainlobe or reduction of sidelobe. In the following the libraries instruction is explained conceptually and then the way of using them to solve MSE problem will be presented in two scenarios using first proposed library as a UPA Library (say UPA-LIB) and the second proposed library as a SLL library (say SLL-LIB), respectively.

A. First Scenario: Beampattern Matching Design with UPA-LIB

Starting from the UPA with M number of antennas that are spaced half wavelength. Beampattern in different directions could be simply achieved with mechanical rotation of elements. Let consider Fig. 1a as an example of mechanical rotation of UPA elements with rotation angle γ and the arc length that we consider equivalent with $\widehat{y}$. According to the geometry of Fig. 1a, $\widehat{y}$ is estimated by $\frac{\lambda}{2}\sin(\gamma)$. If we consider electrical rotation or phase changes in each UPA element as instead of mechanical rotation, therefore, the phase difference for the mth element is equivalent $\Delta\phi = m\frac{2\pi}{\lambda}\widehat{y}$, where $m = 1, \ldots, M$. The electrical rotation of UPA elements is depicted in Fig. 1b. For small value of γ, phase difference equivalents with $\Delta\phi = m\pi\sin(\gamma)$. Let us consider the amplitude and phase of transmitted signal from each element in mechanical rotation as $1\angle 0$ while in electrical rotation it changes to $1\angle m\pi\sin(\gamma)$ for the *m*th antenna element. Next, we could write the covariance matrix based UPA elements structure in Fig. 1b as:

$$\mathbf{R} = \begin{bmatrix} e^{j\pi\sin(\gamma)} \\ \vdots \\ e^{j\pi M\sin(\gamma)} \end{bmatrix} \begin{bmatrix} e^{-j\pi\sin(\gamma)} & \ldots & e^{-j\pi M\sin(\gamma)} \end{bmatrix} \quad (8)$$

It is important to mention that $\mathbf{R}$ in (8) is satisfied in practical constraints (equality of diagonal and PSD of $\mathbf{R}$. The procedure of finding $\mathbf{R}$ in (8), for any number of antennas and for any angle range is very fast (shorter than 10 millisecond in worst case). The instruction of our first proposed library is based on this procedure. It means that for M number of antennas, the proposed facilitator library includes covariance matrixes that are achieved by (8) in a definite range of angle γ . Based on MIMO radar application and its type, the field of view and the desired beampattern can be defined as the range of angle γ in problem input. Let us consider $\{\tilde{\theta}_i\}_{i=1}^{I}$ as the radar's field of view where I indicates the number of angles. We consider any angle in range $\{\tilde{\theta}_i\}_{i=1}^{I}$ as γ therefore, find all the corresponding covariance matrixes by (8) as $\{\tilde{\mathbf{R}}_i\}_{i=1}^{I}$ and store as library members. Next, for M number of antennas, this collection of saved covariance matrixes (as members of UPA-LIB) is used to solve the MSE problem to achieve any desired beampattern. In the proposed technique to solve real-time beampattern matching design problem, we call stored covariance matrixes from UPA-LIB and consider unknown $\{\beta_i\}_{i=1}^{I}$ coefficients as $\{\tilde{\mathbf{R}}_i\}_{i=1}^{I}$ for called covariance matrixes. These coefficients are considered as variables in the proposed optimisation problem. Next, we propose our new covariance matrix as the weighted linear summation of called covariance matrixes as $\sum_{i=1}^{I}\beta_i\tilde{\mathbf{R}}_i$. The goal is to find variables $\{\beta_i\}_{i=1}^{I}$ to find new covariance matrix and reach desired beampattern in presented optimization problem using UPA-LIB as:

$$\min_{\beta_i} \quad \frac{1}{K}\sum_{k=1}^{K}\omega_k[P_d(\theta_k) - a^H(\theta_k)(\sum_{i=1}^{I}\beta_i\tilde{\mathbf{R}}_i)a(\theta_k)]^2 \quad (9)$$

$$s.t \qquad \beta_i \geq 0 \qquad i = 1, \ldots, I$$

where $\omega_k \geq 0$ shows the weight for the kth space grid in $k = 1, \ldots, K$, $\tilde{\mathbf{R}}_i$ is the ith called covariance matrix from UPA-LIB and β_i is the ith corresponding variable. The only constraint for (9) is that the coefficients β_i must be PSD. The equality diagonal elements and PSD constraints for covariance matrix, in comparison with (6), is omitted here because these two practical constraints are already satisfied in UPA-LIB. It means that if $\{\tilde{\mathbf{R}}_i\}_{i=1}^{I}$ meets these two constraints, then the linear summation of multiplication coefficients and called covariance matrixes $\sum_{i=1}^{I}\beta_i\tilde{\mathbf{R}}_i$ would be satisfied. The variables are optimized using the CVX toolbox [35] and this leads to the desired beampattern. The instruction of UPA-LIB and the proposed optimization algorithm for finding $\mathbf{R}$ are illustrated in Fig. 2a and Fig. 2b respectively. The pseudo code for the proposed algorithm of finding t he c ovariance matrix in beampattern matching problem with UPA-LIB is given in Algorithm 1.

Algorithm 1 Proposed algorithm to find covariance matrix in beampattern matching problem by UPA-LIB.

Input: M, $P_d(\theta)$, $\{\tilde{\mathbf{R}}_i\}_{i=1}^{I}$ (from UPA-LIB)
Output: $\mathbf{R} = \sum_{i=1}^{I} \beta_i \tilde{\mathbf{R}}_i$, $P(\theta)$
Consider variables: $\{\beta_i\}_{i=1}^{I}$
for: $i = 1$ to $i = I$ do
    Import $\tilde{\mathbf{R}}_i$ from UPA-LIB
    Calculate $\sum_{i=1}^{I} \beta_i \tilde{\mathbf{R}}_i$
end
Calculate $\{\beta_i\}_{i=1}^{I}$ via (9)
Calculate $\mathbf{R} = \sum_{i=1}^{I} \beta_i \tilde{\mathbf{R}}_i$ and $P(\theta)$ via (4)

We will show in simulation results in section IV that the number of unknown variables $\{\beta_i\}_{i=1}^{I}$ in our technique is considerably lower than other counterpart techniques and it makes our optimization problem to be simple and fast. To prove our claim, we will compare our proposed method in Algorithm 1, with the well-known beampattern matching design problem (SQP method) in [34] where, for M number of antennas, $\frac{M^2-M}{2}$ complex variables must be determined in MSE problem while in our proposed technique, variables $\{\beta_i\}_{i=1}^{I}$ are real and their number is considerably lower than mentioned counterpart. The reducing of the number of variables and consequently the reduction of computational time are more evident when the number of antennas increases.

We will run the algorithm 100 times in simulations in section IV. The time consumed in our first proposed technique is composed of two terms. The first term t_0 is related to the UPA-LIB creation time that is very short, so it can be done simply and fast. The second term t_i is adjusted for applying algorithm in Algorithm 1 in each turn. Therefore, the average computational time is $\sum_{i=1}^{I} \frac{t_0+t_i}{100} \simeq t_i$ .We will show that our proposed technique computational time is significantly lower than its counterpart.

B. Second Scenario: Beampattern Matching Design with SLL Library (SLL-LIB)

In this section according to UPA-LIB, we propose another advanced library as SLL-LIB that is composed of a set of saved covariance matrixes as members. The SLL-LIB members are covariance matrixes that achieved from solving the minimum SLL problem in (7). Let us consider the radar field of view as $\{\hat{\theta}_j\}_{j=1}^{J}$ where J is the number of angles. We state θ_0 in (7) as centre phase in angle range of $\{\hat{\theta}_j\}_{j=1}^{J}$ for M antennas and consider $\theta_2 - \theta_1 = \sin^{-1}(\frac{1}{M})$ and $\Omega = [-90^\circ \sim (\theta_0 - \sin^{-1}(\frac{3}{M}))] \cup [(\theta_0 + \sin^{-1}(\frac{3}{M})) \sim 90^\circ]$. Therefore, we calculate all the corresponding covariance matrixes by (7) and store as $\{\hat{\mathbf{R}}_j\}_{j=1}^{J}$. The collection of these saved covariance matrixes for any number of antennas, create SLL-LIB that is applied to solve the MSE problem. The solution of (7) for different θ_0 is not as fast as UPA-LIB. Therefore, we present SLL-LIB as an offline library to

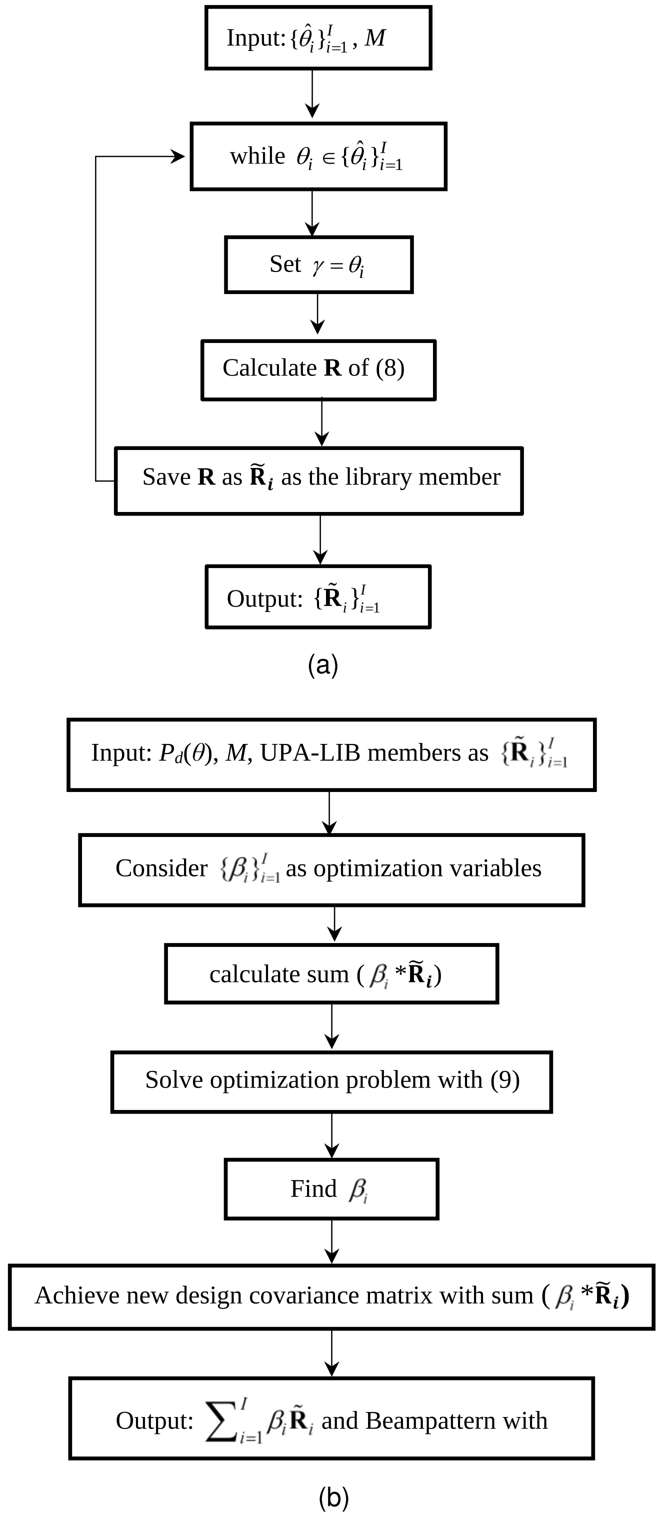


Fig. 2. (a) The instruction of UPA-LIB and (b) the proposed optimization algorithm with UPA-LIB.

save time in any simulation scenario. It means that for every number of antennas, an offline SLL-LIB could be created and then be used in different simulations for solving any real-time beampattern matching design problem. The procedure for solving MSE problem is similar to the beampattern matching design with UPA-LIB. Therefore, we consider a number of unknown variables $\{\eta_j\}_{j=1}^J$ corresponding to the called covariance matrices $\{\hat{\mathbf{R}}_j\}_{j=1}^J$ of SLL-LIB and state the proposed covariance matrix as linear combination of them in form of $\sum_{j=1}^J \eta_j \hat{\mathbf{R}}_j$. These coefficients are considered as unknown variables in the proposed problem. Similar to UPA-LIB, the practical constraints for covariance matrix in (5) are omitted. The formation of new beampattern matching problem can be stated as (10) and solved by CVX.

$$\min_{\eta_j} \quad \frac{1}{K}\sum_{k=1}^{K} \omega_k [P_d(\theta_k) - a^H(\theta_k)(\sum_{j=1}^{J} \eta_j \hat{\mathbf{R}}_j) a(\theta_k)]^2 \tag{10}$$

$$s.t \qquad \eta_j \geq 0 \qquad j = 1, \ldots, J$$

where $\hat{\mathbf{R}}_j$ is the jth called covariance matrix from SLL-LIB and η_j is the jth coefficient. $\sum_{j=1}^J \eta_j \hat{\mathbf{R}}_j$ defined as linear combination of weighted called covariance matrix as our desired final covariance matrix. The only constraint in (10) is that η_j must be PSD. The instruction of SLL-LIB and the proposed optimization algorithm based on SLL-LIB are illustrated in flowchart in Fig. 3a and Fig. 3b respectively. The pseudo code for the second proposed algorithm to find covariance matrix in beampattern matching problem by SLL-Lib is given in Algorithm 2:

Algorithm 2 Proposed algorithm to find covariance matrix in beampattern matching problem by SLL-Lib.

1. **Input**: M, $P_d(\theta)$, $\{\hat{\mathbf{R}}_j\}_{j=1}^J$ (from SLL-LIB)
2. **Output**: $\mathbf{R} = \sum_{j=1}^J \eta_j \hat{\mathbf{R}}_j$, $P(\theta)$
3. Consider variables: $\{\eta_j\}_{j=1}^J$
4. **for**: $j = 1$ to $j = J$ do
5. Import $\hat{\mathbf{R}}_j$ from SLL-LIB
6. Calculate $\sum_{j=1}^J \eta \hat{\mathbf{R}}_j$
7. **end**
8. Calculate $\{\eta\}_{j=1}^J$ via (10)
9. Calculate $\mathbf{R} = \sum_{j=1}^J \eta_j \hat{\mathbf{R}}_j$ and $P(\theta)$ via (4)

IV. Simulation Results

In this section, several numerical simulations are conducted to assess the performance of the proposed algorithm in real-time beampattern matching design problem with the proposed libraries. In all simulations, we assume an array with half-wavelength element spacing and the range of angle to be $[-90° \sim 90°]$ with the resolution of $1°$ which gives $K = 181$ grid points and assume $c = 1$. For all simulations, the desired beampattern is $P_d(\theta)$. Besides, our personal computer that these simulations are conducted on has the configurations of 64-bit Intel i7-8550U CPU and 16GB RAM.

A. Test on Matching Design with UPA-LIB

In this subsection, experiments are conducted to testify the performance of the proposed method using UPA-LIB. For comparison purposes, we consider the well-known counterpart, i.e., the SQP method based on beampattern matching design method in [34] to design the covariance matrix. In the

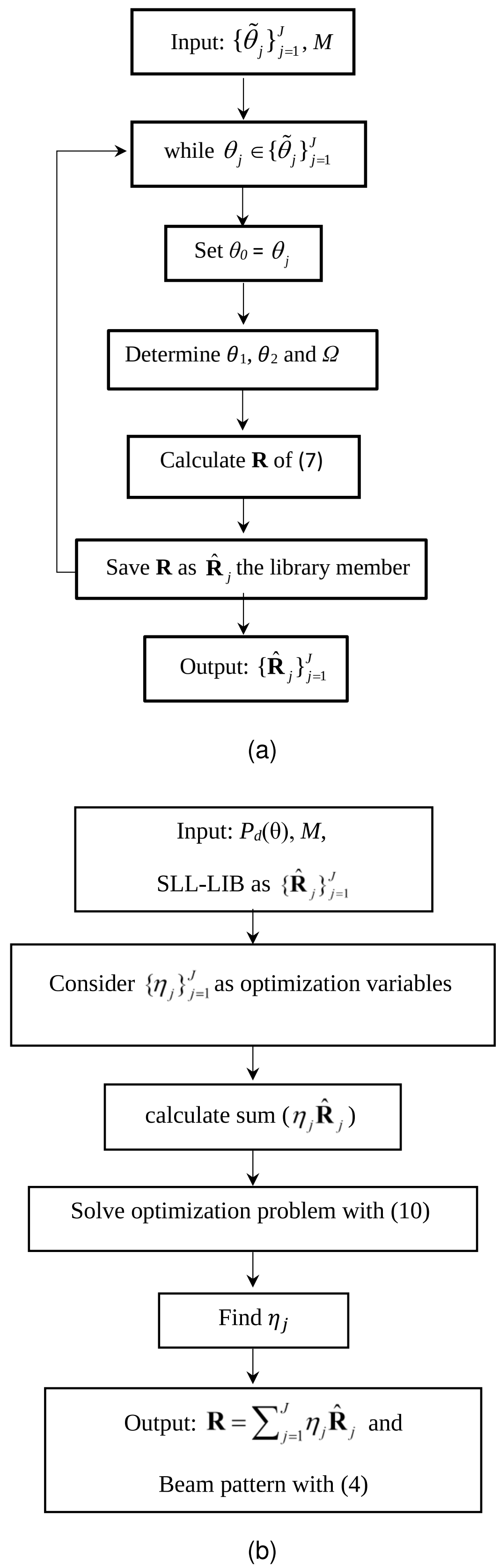


Fig. 3. (a) The instruction of SLL-LIB and (b) the proposed optimization algorithm with SLL-LIB.

first simulation, we consider a desired beampattern matching problem as follows:

$$P_d^1(\theta) = \begin{cases} 1 & \theta \in \{[-20, -10] \cup [10, 20]\} \\ 0 & otherwise, \end{cases} \quad (11)$$

which can be considered as two mainlobes with centres at $\theta = -15°$ and $\theta = 15°$ and each beamwidth is $10°$. Also, we considered $M = 16$ and $M = 30$. Fig. 4 shows the results of the first simulation for the proposed method with UPA-LIB as a comparison with the stated counterpart. As can be seen, our proposed method has lower sidelobe levels and the same mainlobes ripples.

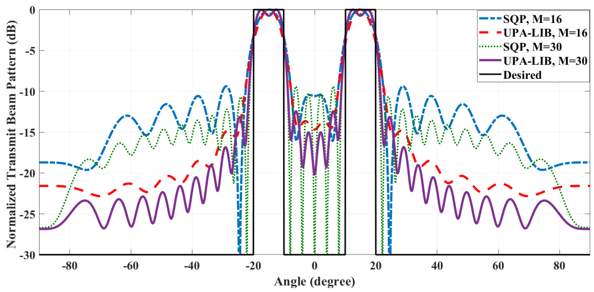


Fig. 4. Normalized comparison beampattern design results in Simulation 1.

Furthermore, comparison of the runtimes and number of unknown variables of two algorithms in two different number of antennas are provided in Table I. Comparatively, these parameters in the UPA-LIB-design algorithm change are considerably lower than SQP. All those imply that the proposed algorithm is more applicable especially to large number of antennas. It is important to mention that the unknown variables in SQP method are complex (in asymmetric beampattern design), while in the presented method, the coefficients are real, and this is another advantage of our proposed method.

TABLE I
COMPARISON OF RUNTIMES IN SIMULATION. 1 AND THE NUMBER OF UNKNOWN VARIABLES

	Runtimes (seconds)		Number of unknown variables	
Method	UPA-LIB	SQP	UPA-LIB (real)	SQP (complex)
$M = 16$	2.3	5.16	41	120
$M = 30$	2.4	56.7	41	435

For the second simulation, we consider synthesizing an asymmetric desired beampattern is defined as follows:

$$P_d^2(\theta) = \begin{cases} 1 & \theta \in \{[-47, -44] \cup [9, 12] \cup [40, 60]\} \\ 0 & otherwise, \end{cases} \quad (12)$$

which can be considered as three mainlobes with centres at $\theta = -45.5°$, $\theta = 10.5°$ and $\theta = 50°$ and each beamwidth is $3°$, $3°$ and $20°$. The number of antennas considered $M = 20$ and $M = 30$ number. Fig. 5 shows the results of the second simulation for the proposed method in two different number of antennas for SQP and desired beampattern. It is shown that our method synthesizes a beampattern which matches to the desired beampattern better than SQP with providing lower sidelobe levels (around 10dB) in region between two narrow band mainlobes.

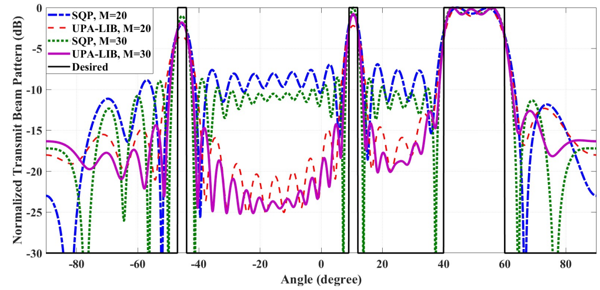


Fig. 5. Normalized comparison beampattern design results in Simulation 2.

TABLE II
COMPARISON OF RUNTIMES IN SIMULATION. 2 AND THE NUMBER OF UNKNOWN VARIABLES

	Runtimes (seconds)		Number of unknown variables	
Method	UPA-LIB	SQP	UPA-LIB (real)	SQP (complex)
$M = 20$	2.78	12.5	108	190
$M = 30$	3.26	49.17	108	435

Comparison of the runtimes and number of unknown variables of the proposed method and SQP are shown in Table II. Significantly decrease of these parameters in our method rather than SQP is evident. As can be seen, in high number of antennas, the number of unknown variables in our proposed method is approximately a quarter of SQP and the run time is decreased around 93%.

In the simulations 3 and 4, we consider a desired beampattern matching problem for orthogonal MIMO and simple narrow band phase array at $\theta = 0°$ as (13) and (14) respectively:

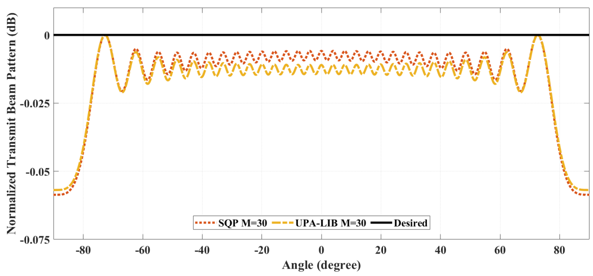
$$P_d^3(\theta) = \begin{cases} 1 & \theta \in \{[-90, 90]\} \\ 0 & otherwise, \end{cases} \quad (13)$$

$$P_d^4(\theta) = \begin{cases} 1 & \theta \in \{[-1, 1]\} \\ 0 & otherwise, \end{cases} \quad (14)$$

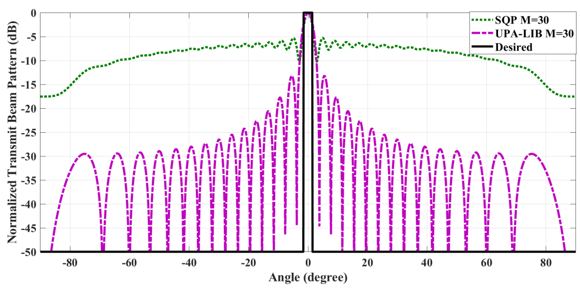
In these two simulations, we consider $M = 30$. Fig. 6a and Fig. 6b show the results of this simulation for the proposed method with UPA-LIB as well as stated counterpart for orthogonal MIMO and narrow linear phase array, respectively. As can be seen, our proposed method has lower sidelobe levels in narrow linear phase array and the same mainlobe ripple in MIMO orthogonal. According to Table III, by significantly reducing the calculation time and greatly reducing the number of unknown variables, especially in Simulation 4, we have been able to reach the usual answers in the literature [10].

TABLE III
COMPARISON OF RUNTIMES IN SIMULATION. 3 AND 4 THE NUMBER OF UNKNOWN VARIABLES

	Runtimes (seconds)		Number of unknown variables	
Method	UPA-LIB	SQP	UPA-LIB (real)	SQP (complex)
UPA (in $\theta = 0°$)	1.3	35.4	3	435
Orthogonal MIMO	3.7	30.5	181	435



(a)



(b)

Fig. 6. (a) Normalized comparison beampattern design results in MIMO Orthogonal in Simulation 3 and (b) narrow band phase array in Simulation 4.

As it can be seen in orthogonal MIMO with maximum beamwidth, the number of variables is less than half of SQP. The number of variables in the proposed method will decrease by beamwidth reduction.

In order to gain a better insight of the efficiency of our proposed method in a more complex beampattern scenario with a large number of antennas, we consider a beampattern matching problem in the simulation 5 as follows:

$$P_d^5(\theta) = \begin{cases} 1 & \theta \in \{[-40,-35] \cup [-25,-15] \cup [-5,5] \\ & \cup [15,20] \cup [40,50]\} \\ 0 & otherwise \end{cases} \tag{15}$$

which can be considered as five asymmetric mainlobes with centres at $\theta = -37.5°$, $\theta = -20°$, $\theta = 0°$, $\theta = 17.5°$ and $\theta = 45°$. The number of antennas considered $M = 30$ and $M = 50$. Fig. 7 shows the results of the fifth simulation for the proposed method with UPA-LIB as well as the stated counterpart. As reflected in Fig. 7, the proposed method's beampattern has comparable ripple in the mainlobes region and lower sidelobe levels relative to SQP. Also, the runtimes and number of unknown variables in the optimization problem are provided in Table IV .As seen earlier, the runtime and number of unknown variables of the UPA-LIB-design algorithm change significantly and is considerably lower than that of SQP especially to large-number of antennas. It is obvious from Table IV that the number of unknown variables is reduced 92% in $M = 50$ case which considerably increases the speed of solving the corresponding optimization problem.

B. B. Test on Beampattern Matching Design with SLL-Lib

In this subsection, we simulate the performance of the second proposed algorithm using SLL-Lib. For comparison

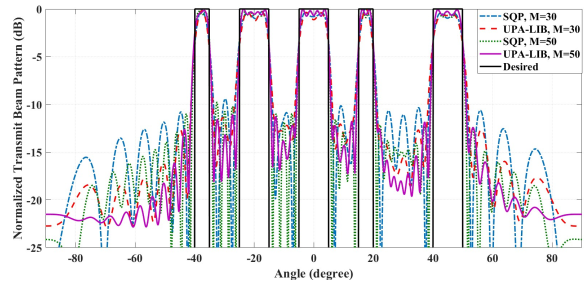


Fig. 7. Normalized comparison beampattern design results in Simulation. 5

TABLE IV
COMPARISON OF RUNTIMES IN SIMULATION. 5 AND THE NUMBER OF UNKNOWN VARIABLES

	Runtimes (seconds)		Number of unknown variables	
Method	UPA-LIB	SQP	UPA-LIB (real)	SQP (complex)
$M = 30$	3.9	57.7	91	435
$M = 50$	5.39	544	91	1225

purposes, we implement the SQP method and our first proposed method with UPA-LIB. In the following simulations we created libraries for $M = 6, 10$ and 25 number of antennas (based on the second scenario in section III) and used of them offline. In the simulation 6, the desired beampattern is considered as follows:

$$P_d^6(\theta) = \begin{cases} 1 & \theta \in \{[5,20]\} \\ 0 & otherwise, \end{cases} \tag{16}$$

which can be considered as one mainlobe with centres at $\theta = 12.5°$ and beamwidth is $\theta = 15°$. The number of antennae considered $M = 6$. Fig. 8 shows the results of this simulation for the proposed method with SLL-LIB, UPA-LIB as well as stated counterpart. As can be seen, this method has lower sidelobe levels and the same mainlobe ripple compared to the SQP and UPA-LIB.

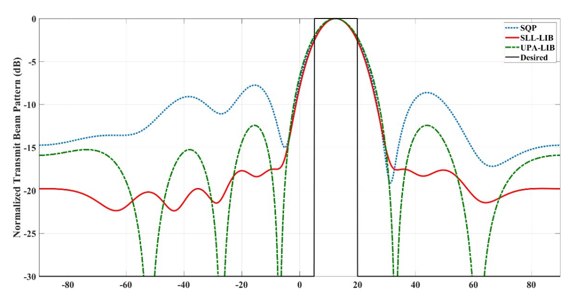


Fig. 8. Normalized comparison beampattern design results in Simulation 6 $M = 6$.

In the simulation 7, the wide band and symmetric desired beampattern is considered as follows:

$$P_d^7(\theta) = \begin{cases} 1 & \theta \in \{[-30,30]\} \\ 0 & otherwise, \end{cases} \tag{17}$$

which can be considered as wide symmetric beampattern with centres at $\theta = 0°$ and beamwidth is $\theta = 60°$. The number of antennae is considered $M = 10$. Fig. 9 shows the results of the simulation for the proposed method with UPA-LIB and SLL-LIB as well as stated counterpart.

As can be observed, the performance improvement of the provided enhanced library is considerable. As the last scenario

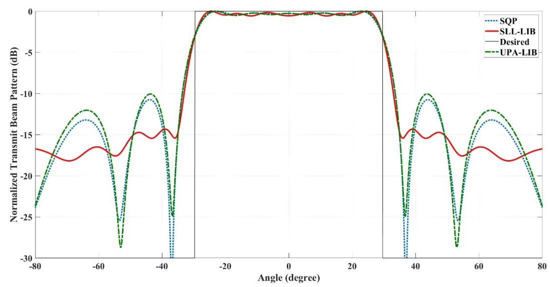


Fig. 9. Normalized comparison beampattern design results in Simulation 7 $M = 10$.

in the simulation 8, the desired beampattern is considered as follows:

$$P_d^8(\theta) = \begin{cases} 1 & \theta \in \{[-30, -15] \cup [15, 30]\} \\ 0 & otherwise, \end{cases} \quad (18)$$

which can be considered as two mainlobes with centres at $\theta = -22.5°$ and $\theta = 22.5°$ and each beamwidth is $\theta = 15°$. The number of antennas is considered $M = 25$. Fig. 10 shows the comparison results. In the proposed method with the SLL-LIB, we can reduce the side level while achieving the appropriate level of the mainlobes. Furthermore, the runtimes

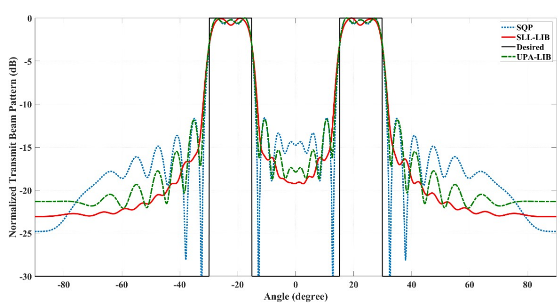


Fig. 10. Normalized comparison beampattern design results in Simulation 8 $M = 25$.

and the number of unknown variables for the simulation 6, 7 and 8 are provided in Table V. Comparatively, the runtime of the optimization algorithm for SLL-LIB-design is lower than SQP. All those imply that the proposed algorithm is more applicable especially for large number of antennas. Additionally, the number of unknown variables of the SLL-LIB design algorithm is equivalent with UPA-LIB beampattern matching design that reduce considerably 80% than SQP.

V. Conclusion

A novel approach for covariance matrix design in collocated MIMO radar has been presented. The innovation of this approach is to solve the beampattern matching problem using facilitator libraries as UPA-LIB and SLL-LIB. We proposed a new covariance matrix design technique using the facilitator library members in a reduced number of unknown variables in MSE problem and therefore in a significantly lower time. We showed in different scenarios and simulation cases that this technique outperforms its well-known counterpart (SQP) in terms of computational complexity and consumed time, while keeping an acceptable beampattern matching level, thus making the system more affordable for real-time scenarios. The proposed technique is a straightforward algorithm with high speed which makes it an appropriate and practical method for real-time applications where fast beamforming for a large number of real-time beampatterns in different angles and large number of antennas, is essential. This can have various applications in future researches such as automated driving systems, health monitoring structure, displacement measurements and mimo radar image reconstruction, where MIMO radar based high-speed beamforming is required.

TABLE V
Comparison of runtimes in simulation. 2 and the number of unknown variables

	Runtimes (seconds)			Number of unknown variables		
Method	SLL-LIB	UPA-LIB	SQP	SLL-LIB (real)	UPA-LIB (real)	SQP (complex)
$M = 6$	1.9	1.75	1.66	21	21	15
$M = 10$	1.98	2.15	5.65	61	61	120
$M = 25$	2.1	2.2	23.7	61	61	300

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