Predictive control co-design for enhancing flexibility in residential housing with battery degradation ⋆

P. Falugi * **E. O'Dwyer** ** **E. C. Kerrigan** *** **E. Atam** *
M. A. Zagorowska * **G. Strbac** * **N. Shah** **

* *Department of Electrical and Electronic Engineering, Imperial College London, London, UK (e-mails: p.falugi@imperial.ac.uk, e.atam@imperial.ac.uk, m.zagorowska@imperial.ac.uk, g.strbac@imperial.ac.uk).*
** *Department of Chemical Engineering, Imperial College London, London, UK (e-mails: e.odwyer@imperial.ac.uk, n.shah@imperial.ac.uk)*
*** *Department of Electrical and Electronic Engineering, Department of Aeronautics, Imperial College London, London, UK, (e-mail: e.kerrigan@imperial.ac.uk)*

Abstract: Buildings are responsible for about a quarter of global energy-related CO_2 emissions. Consequently, the decarbonisation of the housing stock is essential in achieving net-zero carbon emissions. Global decarbonisation targets can be achieved through increased efficiency in using energy generated by intermittent resources. The paper presents a co-design framework for simultaneous optimal design and operation of residential buildings using Model Predictive Control (MPC). The framework is capable of explicitly taking into account operational constraints and pushing the system to its efficiency and performance limits in an integrated fashion. The optimality criterion minimises system cost considering time-varying electricity prices and battery degradation. A case study illustrates the potential of co-design in enhancing flexibility and self-sufficiency of a system operating under different conditions. Specifically, numerical results from a low-fidelity model show substantial carbon emission reduction and bill savings compared to an a-priori sizing approach.

Keywords: Modeling for control optimization, Energy systems, Linear parameter-varying systems, Interaction between design and control

1. INTRODUCTION

Achieving net-zero carbon emissions by 2050 will entail significant changes to the way electrical energy is generated, transmitted and used. Leveraging operational flexibility will be a critical challenge for developing a cost-efficient net-zero carbon system. Currently, technologies that support the functionality of residential buildings, such as thermal solar, PV panels and heating systems, are sized without considering how they perform in conjunction with the Building Management System (BMS) that will operate them. In this work, we present an approach to sizing the optimal technology mixes, while incorporating the effect on the operation of the optimised system. The results are presented in a case study for residential buildings equipped with battery storage systems, subject to degradation.

Simultaneous optimisation of design and operation of a system is called *co-design*. Diangelakis et al. (2017) provides an overview of control approaches used in co-design for process systems engineering applications. The need for increased flexibility has sparked interest in optimal system operation and configuration at the residential building level. Optimal design approaches for battery sizing with photo-voltaic panels (PV) and energy management of smart homes have been proposed by (Wu et al., 2017) and (Beck et al., 2016). (Koskela et al., 2019) further emphasized the impact of the size of PVs and battery on the profitability from the economic perspective. In particular, (Koskela et al., 2019) analysed the interactions between the size of the PVs and battery storage, but without considering the impact of battery degradation on the operation of the system. (Sorourifar et al., 2020) have shown that battery degradation leads to decreased performance of energy management systems and proposed a strategy for replacing degraded batteries. However, their solution focused on economic aspects of energy management with battery storage systems. The present contribution includes a detailed case study on optimal sizing of battery storage and the roof area used by photo-voltaic panels (PV), while considering optimal management of the building assets in the form of a single finite-horizon optimal control problem.

The paper is structured as follows. Section 2 introduces the model of the considered residential building. Section 3 presents the co-design framework for residential buildings,

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Table 1. Dwelling parameters

Description	Parameter	Value	Unit
Average U-value	U	0.93195	W/(m^2 K)
Wall surface area	A	82.06959707	m^2
Air density	ρ_{air}	1.225	kg/m^3
Building volume	V	224.05	m^3
Air heat capacity	C^p_{air}	1.005	kJ/(kg K)
Air changes per hour	n_{ac}	1	h^{-1}
Building thermal mass	C_{build}	15286.6114	kJ/K
Floor surface area	S_F	89.62	m^2
HP electricity bound	$\overline{u}^{eH}$	4	kW
CP electricity bound	$\overline{u}^{ceH}$	6	kW
HP capacity	$\overline{Q}^{HP}$	6	kW

while Section 4 defines the case study and illustrates and discusses the mutual influences between design and predictive control. Finally, the conclusions are presented in Section 5.

2. BUILDING MODEL

An essential feature of future residential buildings is the integration of storage assets with on site generation capacity to enhance self-sufficiency, resilience and flexibility of the building. The thermal mass of the dwelling can be effectively modelled by adopting the widely used single-zone lumped-capacitance method as proposed in (Hazyuk et al., 2012). The parameters of the building thermal model are reported in Table 1. They represent the physical properties of a well insulated building. The building internal temperature T_t is described by the first-order ordinary differential equation

$$C_{build}\frac{dT_t}{dt} = M(T^e_t - T_t) + Q^{HP}_t - Q^{CP}_t, \tag{1}$$

where $M := UA+\rho_{air}VC^p_{air}n_{ac}$, T^e_t is the outdoor temperature and the heat regulation is provided by heat pumps (HP). In particular, Q^{HP}_t and Q^{CP}_t are, respectively, the heat and cooling provided by electrically driven heat pumps. Comfort requirements, according to standards in (CIBSE, 2015), are imposed through constraints on the internal temperature of the building as

$$\underline{T}_t \leq T_t \leq \overline{T}_t, \tag{2}$$

where $\underline{T}_t$ and $\overline{T}_t$ are the desired thermal comfort limits.

The heat pump extracts heat from ambient air and its key modelling aspect is its Coefficient of Performance (COP), which is the ratio between supplied heat Q^{HP}_t and the consumed electric power u^{eH}_t

$$Q^{HP}_t = \mathrm{COP}(T^e_t)u^{eH}_t. \tag{3}$$

Since the output of a heat pump decreases with external temperature, in the present study we have modelled the COP as a linear function of the external temperature:

$$\mathrm{COP}(T^e_t) = m_{\mathrm{COP}}(T^e_t - 7) + 3. \tag{4}$$

The cooling pump coefficient COP_{cool} has been assumed constant. The values of parameters are listed in Table 2. The symbols T and I_r denote the temperature and the irradiance, respectively.

The building is equipped with a rechargeable lithium battery to enhance system flexibility and adaptability. A model of the battery state of charge SoC_t is

$$\dot{SoC_t} = \eta^{ch}u^{ch}_t - \frac{u^{dch}_t}{\eta^{ds}}, \tag{5}$$

Table 2. Asset parameters

Description	Parameter	Value	Unit
HP COP slope	m_{COP}	0.067	$^\circ$ C
CP COP	COP_{cool}	0.7	-
Battery charging	η^{ch}	0.88	-
Battery discharging	η^{ds}	0.88	-
Max battery size	$\overline{SoC}$	60	kWh
Power/(I_r) gain	θ_1	0.12	kW/m^2
Power/(I_r) correction	θ_2	$-1.345e^{-4}$	-
Power/(T I_r) correction	θ_2	$-3.25e^{-3}$	-

where u^{dch}_t and u^{ch}_t denote the discharging and charging rates, respectively. The parameters η^{ch} and η^{ds} model the charging and discharging efficiencies. The capacity and charge/discharge limits are

$$\begin{aligned} 0 \leq SoC_t \leq S^B \\ 0 \leq u^{dch}_t, u^{ch}_t \leq S^B/T_{ds} \end{aligned} \tag{6}$$

where T_{ds} represents the number of hours required to fully discharge a battery at the maximum discharge rate.

The degradation of a battery strongly depends on how it is operated. Degradation consists in the incremental charging capacity loss of the battery during its lifetime operation. (Fortenbacher and Andersson, 2017) modelled battery degradation as a function of the battery inputs, its state of charge and the capacity.

The degradation process can be described with piecewise affine maps

$$d_t = \max_{k=1,\ldots,n_s}\left\{a_{1,k}(u^{dch}_t + u^{ch}_t) + a_{2,k}SoC_t + a_{3,k}S^B\right\} \tag{7}$$

with constant coefficients $a_{1,k}$, $a_{2,k}$, $a_{3,k}$ $\in \mathbb{R}$ for all $k = 1,\ldots,n_s$. The degradation map is included in an optimization framework by introducing an additional input u^d_t required to satisfy the constraint $u^d_t \geq d_t$. The capacity loss affects the battery value, and its degradation is related to the overall battery cost. The vector parameters a_1, a_2, a_3 for the battery in this study are from (Fortenbacher and Andersson, 2017).

The present case study includes the option of installing PV panels. The maximum power produced by PV panels can be modelled as a nonlinear function (Dows and Gough, 1995; Pepe et al., 2018) of the solar irradiance I_t and the external temperature

$$P^{PV}_t = \theta_1(1 + \theta_2 I_t + \theta_2 T^e_t)I_t S^{PV}, \tag{8}$$

where S^{PV} denotes the roof area covered by the PV panel and θ_i, $i = 1,\ldots,3$ are constant parameters. The operating limits refer to the design specs for the multi-crystalline JAP6 4BB module range manufactured by JA (JA SOLAR Technology Co.,Ltd., 2020). The parameter values for a location in the south of the UK are reported in Table 2.

3. OPTIMAL CO-DESIGN FRAMEWORK FOR RESIDENTIAL BUILDINGS

The sizing problem requires consideration of how the system is operated for at least a year. The proposed co-design problem minimises an economic cost comprising a time varying electricity bill, carbon emissions and annualised capital cost of the battery and PV panels. The problem is required to satisfy operational and physical constraints and is defined as

$$\min_{\substack{u,x,\\S^B,S^{PV}}} \int_{t_0}^{t_f} \ell(x_t,u_t,t)\mathrm{d}t + C^{PV}S^{PV} + C^B S^B \tag{9}$$

$$\text{subject to } (1),(2),(3),(4),(5),(6),(8) \tag{10}$$

$$u_t^B - u_t^S + u_t^{dch} - u_t^{ch} + P_t^{PV} = u_t^{eH} + u_t^{CeH} \tag{11}$$

$$0 \le u_t^B \le 30,\ 0 \le u_t^S \le 30 \tag{12}$$

$$0 \le u_t^{eH} \le \overline{u}^{eH},\ 0 \le u_t^{CeH} \le \overline{u}^{CeH} \tag{13}$$

$$0 \le S^{PV} \le S_F, \tag{14}$$

$$0 \le S^B \le \overline{SoC} \tag{15}$$

$$\mathbf{1}u_t^d \ge \mathbf{a_1}(u_t^{dch} + u_t^{ch}) + \mathbf{a_2}SoC_t + \mathbf{a_3}S^B \tag{16}$$

for all $t \in [t_0, t_f]$, where $x : \mathbb{R} \to \mathbb{R}^3$ and $u : \mathbb{R} \to \mathbb{R}^7$ denote the state and control input trajectories of the system, $\ell(x_t, u_t, t) := c_t^p u_t^B + c^d u_t^d - c_t^S u_t^S$, $\mathbf{a_1}$, $\mathbf{a_2}$, $\mathbf{a_3}$, $\mathbf{1} \in \mathbb{R}^{n_s}$, u_t^B and u_t^S are the bought and sold power, respectively, with a bound of 30kW determined by the connection contract. The inequality (16) consists of 18 constraints with coefficients provided by (Fortenbacher and Andersson, 2017). The optimal u_t^{d*} corresponds to the capacity loss d_t from equation (7) induced by the optimal management of the battery. The physical limitation that the total area occupied by the PV panels must not exceed the available roof area is expressed by (14). The capacity S^B of the battery is limited by the constraint (15).

Boundary cyclic conditions (BC) are included as

$$x(t_0) = x(t_f) \tag{17}$$

to enforce the periodic steady-state optimal solution to problem (9). A periodic steady-state trajectory can be thought of as a repeatable optimal operating condition in the subsequent years and defines an MPC formulation on an infinite horizon for the deterministic case. Thus, the final time t_f was set to one year to capture the periodic behaviour of a building. The sensitivity of the solution with respect to the boundary conditions (17) will be investigated to analyse its effective impact in a problem presenting extremely fast dynamics with respect to the periodicity under consideration. Note that, since we consider a deterministic formulation of the sizing problem through the whole prediction horizon with periodic steady-state conditions, for the certainty equivalence principle, it is not necessary to use a feedback formulation.

The prices $c_t^p := c_t^B + c_t^m$ include the electricity prices c_t^B and the price c_t^m of carbon emissions. The studies have been performed considering time-varying electricity prices c_t^B with 15 minutes resolution based on Nordpool data with a pricing mechanism used as described by Octopus Tracker (Octopus Energy Ltd., 2020). It has been assumed that at each time instant the price of the sold power c_t^S satisfies $c_t^S = 0.9c_t^B$. Meteorological data profiles for different UK locations have been obtained from a database maintained by the European Commission Joint Research Centre (JRC, 2012). To analyse the benefits of system design with consideration of the operational strategy and objective choice, we have solved the problem under different conditions and investment costs (see Table 3). Furthermore, an additional case has been created in which technologies are sized without consideration of how the building is operated. In particular, following the current practice, PV panels have been chosen to cover

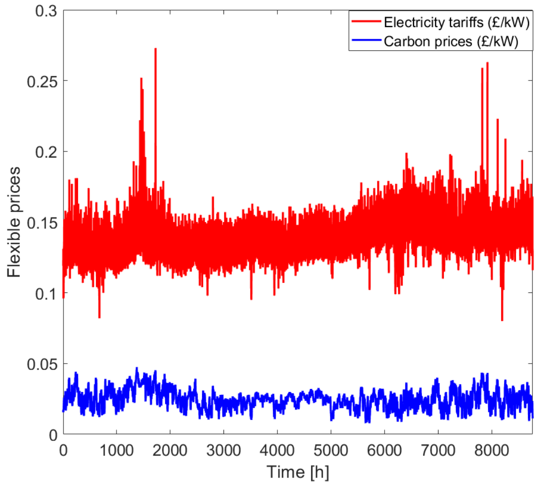

Fig. 1. The costs for carbon emissions and electricity prices throughout the whole year considering a carbon price cost of 100£/(ton CO2e)

Table 3. Asset Investment Parameters

	C^{PV}	C^B	c^d
CAPEX_1	80£/m^2	10£/kWh	10£/kWh
CAPEX_2	80£/m^2	0£/kWh	500£/kWh
CAPEX_3	300£/m^2	500£/kWh	500£/kWh
Technology lifespan (years)	30	15	-

an area of $S^{PV} = 44.81\text{m}^2$ jointly with a battery of 30kWh capacity. The capital costs, referred to as CAPEX, used in the reported results are provided in Table 3 and the degradation cost is given by the battery cost. The annualised capital costs have been computed considering the technology lifespan in Table 3 and an interest rate $r = 2\%$. In particular, the equivalent annual cost of each technology is obtained dividing the capital cost (CAPEX) by the "present value of annuity factor"

$$a_{y,r} = \frac{1 - \frac{1}{(1+r)^y}}{r}$$

with y denoting the year.

4. CASE STUDY

The co-design problem (9) has been solved using the investment costs in Table 3 and analysed under different operating assumptions. In particular, the impact of the degradation model and the inclusion of boundary conditions have been investigated. The analysis illustrates the benefits of adopting a co-design framework in enhancing the performance of a system operating under varying external conditions through the whole year. The co-design problem (9) has been implemented in Matlab using ICLOCS2 (Nie et al., 2018) and solved with IPOPT (Wächter and Biegler, 2006). The studies have been performed on a laptop equipped with an Intel(R) Core(TM) i7-9850H CPU @ 2.60GHz processor. The adopted transcription method is the explicit Euler with 15 minutes time resolution. The Euler method enables analysis of systems with discontinuities in the inputs. The maximum time resolution is dictated by the piece-wise constant electricity prices which update every 15 minutes. The technol-

Table 4. Sensitivity of the solution versus the boundary conditions

Case CAPEX1	Problem (9) with BC	Problem (9) No BC
Battery Capacity (kWh)	7.88	11.87
Area PV (m^2)	89.62	89.62
Operational Cost (£/y)	−802.83	−807.78
Investment Cost (£/y)	326.25	329.36
Total Optimal Cost (£/y)	−476.58	−478.42
Net Carbon emissions (Kg/y)	−1610.9	−1622.2

Table 5. Solutions with extreme CAPEX

Case	Problem (9) $CAPEX_2$ + BC	Problem (9) $CAPEX_3$ + BC
Battery Capacity (kWh)	60	0
Area PV (m^2)	89.62	89.62
Operational Cost (£/y)	−825.25	−793.03
Investment Cost (£/y)	320.12	1200.5
Total Optimal Cost (£/y)	−505.61	407.43
Net Carbon emissions (kg/y)	−1693.8	−1606.5

ogy sizes and the costs obtained solving the optimization problem (9)-(16) with and without Boundary Conditions (BC) adopting $CAPEX_1$ are reported in Table 4. Since the incremental variations in the electricity prices are relatively small, the solution demonstrates sensitivity to the inclusion of the boundary conditions even if the horizon covers the whole year.

Table 5 compares the optimal solutions employing $CAPEX_2$ and $CAPEX_3$. $CAPEX_3$ uses realistic capital costs. The capital cost of the battery is too high to be economically advantageous. The optimal decision consists of installing the maximum possible PV generation capacity to immediately sell or use the generated energy and mitigate the bill expenses. Conversely, using $CAPEX_2$, since, in the objective, the variable electricity prices dominate the contribution of the expenses, the co-design framework recommends installing technologies of maximum possible size. In this case, flexibility is precious since the variable electricity prices describe the grid's flexibility requirements aside from determining the electricity bill of the building owner. Indeed the problem under consideration is a multi-objective problem, and the optimal solution depends on the relative importance of its terms. In other words, small capital costs can be interpreted as giving priority to flexibility. Note that, in the studies reported in Table 5, the magnitude of the operational cost and the net carbon emissions are similar. The Net Carbon Emissions represent the emission variation contributed by the active building across one year. The emission index accounts for a positive contribution due to purchased electricity and a negative one accrued by selling electricity.

Note that the solution obtained for $CAPEX_3$ excludes the storage technologies, but its performances in terms of operational cost and net carbon emissions are comparable to those obtained using $CAPEX_1$ and $CAPEX_2$ under the same operating conditions. However, the performance of the design that excludes storage assets may deteriorate considerably under different operating conditions. This situation happens because, in the cost, the value of flexibility is not precisely quantified. Also, the occurrence of different future scenarios is ignored.

Table 6. Performances under different assumptions

Case	Pre-sized + BC	Problem (9) no sell $CAPEX_1$ + BC
Battery Capacity (kWh)	30	8.86
Area PV (m^2)	44.81	36.10
Operational Cost (£/y)	−182.31	240.06
Investment Cost (£/y)	−	135.86
Total Optimal Cost (£/y)	−	375.92
Net Carbon emissions (Kg/y)	−444.47	394.14

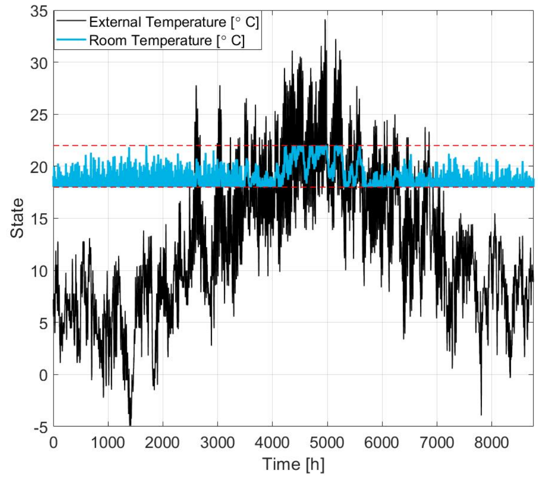

Fig. 2. Temperatures

The first column in Table 6 reports the annual operational cost achieved by a predictive controller with predetermined technologies. The model predictive controller is based on the problem (9)-(16) assigning the technologies' size determined a-priori. The bill savings and the net carbon emission reductions are substantially inferior compared to all the case studies using the co-design framework with the option of providing energy to the grid.

The optimal performances and the technology mix of a residential building without the option of selling power to the grid are documented in the second column of Table 6. This latter case shows lower PV capacity and a slightly increased battery capacity than the corresponding problem with the possibility of selling energy to the grid in Table 4. The result is due to the fact that the local generated energy is substantially higher in summer than in winter and any excess energy is wasted.

The achieved performances of the building at the operational level are illustrated for the case study $CAPEX_1$ with the inclusion of boundary conditions. The optimisation problem has been solved in about 2h 20 m. Figure 2 shows how the predictive controller satisfies the thermal comfort requirements over the year while the left thermal margins provide energy storage capability reducing the size of the needed battery. Figures 3 and 4 illustrate the management of the battery in a summer week in the $CAPEX_1$ and pre-sized cases, respectively. In the pre-sized case, the battery is underutilised, and it is oversized for the considered operating conditions. Conversely, the battery obtained with the co-design process are fully and efficiently used through the whole year.

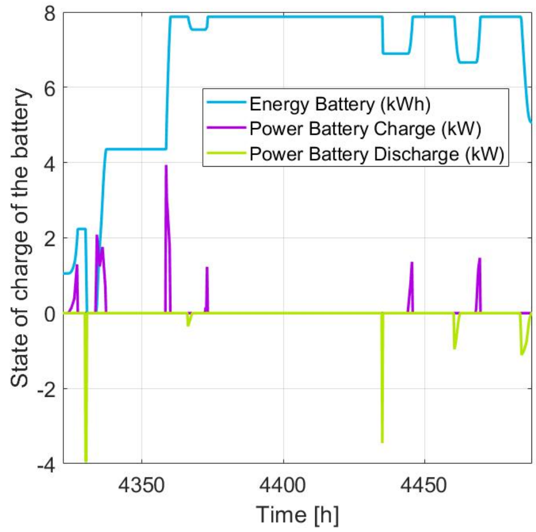

Fig. 3. Battery behaviour in a summer week ($CAPEX_1$ with BC)

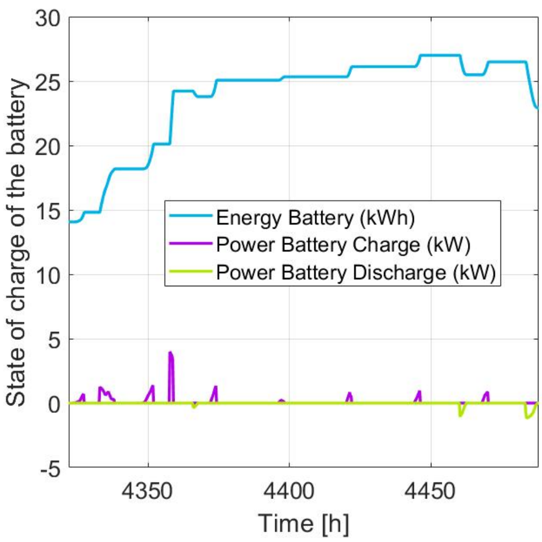

Fig. 4. Battery behaviour in a summer week (pre-sizing with BC)

The electricity consumption of the heat and cooling pumps obtained with the co-design approach are shown in Figure 5. Conversely, the power profiles in Figures 6 and 7 are substantially different. The oversized battery in the pre-sized case trades larger quantities of energy with the grid causing higher peaks of bought and sold power.

The importance of including a battery degradation model is demonstrated in Figure 8, which reports the energy and power charge/discharge profiles of the battery with and without the degradation model. The results highlight how the battery is subject to a more intensive use in the case where the degradation is not considered.

Note that, since the degradation process consists of a loss capacity effect, the sensitivity of the optimal solution with respect to the degradation process increases with its

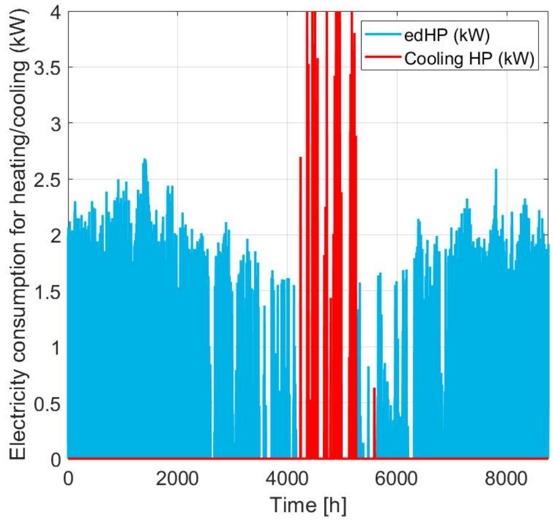

Fig. 5. Electricity consumption for thermal comfort ($CAPEX_1$ with BC)

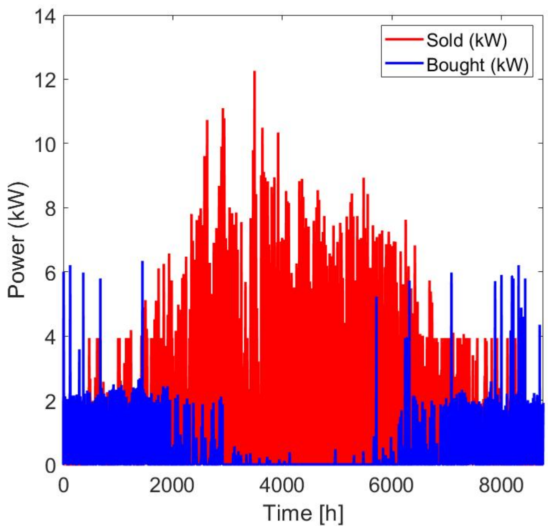

Fig. 6. Bought and sold power ($CAPEX_1$ with BC)

capital cost. Once the battery capacity fades, there is a cost due to its replacement.

5. CONCLUSIONS

The achievement of net-zero carbon emissions requires decarbonisation of the entire housing stock. We have presented an approach to simultaneously optimize the design and the operation of residential buildings considering external weather conditions and time-varying electricity prices. A case study demonstrates the ability of the presented co-design framework to seek trade-offs in an integrated fashion with a temporal resolution spanning from years to minutes. The results show a sensitivity of the optimal solution to cyclic constraints accounting for the possible yearly seasonal periodicity. The phenomenon appears for the relatively low available incremental prices and is due to a lack of a systematic way of pricing the initial and final state of the system. This high sensitivity also demonstrates that the range of price variations is such

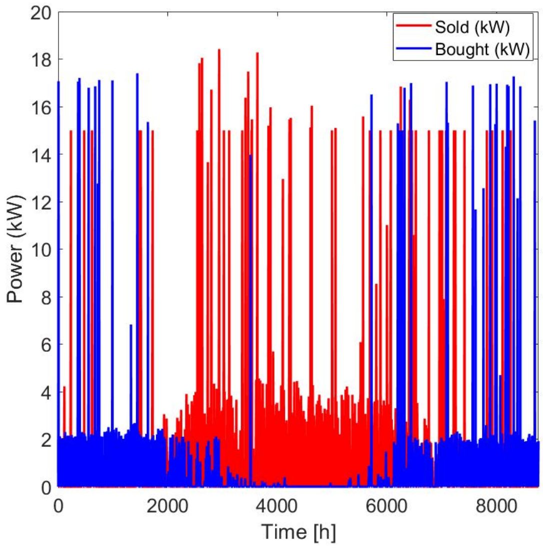

Fig. 7. Bought and sold power (pre-sizing with BC)

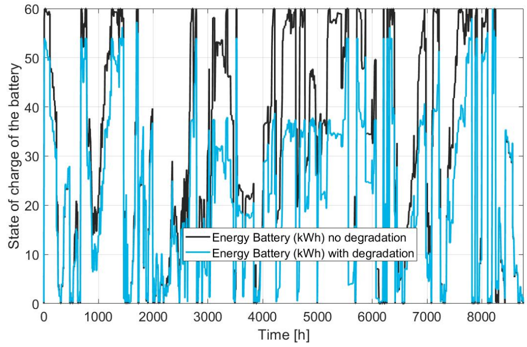

Fig. 8. Battery optimal management with and without degradation model for $CAPEX_3$

that the value of the initial energy stored is comparable to the saving achieved. It also suggests that the combination of the considered technologies is only convenient in highly dynamic electricity markets unless other sources of revenue, such as ancillary services, are accounted for as possible additional income.

In particular, the case study based on a low-fidelity model reported for the co-design framework quadrupled emission reductions. Simultaneous design and control leads to a total annual bill saving of up to £475 compared to the a-priori sizing approach.

Overall, the flexibility of MPC and optimal design of residential buildings indicate that the presented framework is a good candidate for future work, such as data-driven control and robust optimization. The case study could be further explored, taking into account an improved accuracy of the model and the effect of the uncertainty to improve the understanding of the interplay between design and operation of a system. Analysis of the value of flexibility against potential cost savings, carbon emission reductions, and the robustness against uncertainties might enable better handling of uncertainty inherent in real-life applications. Future work could also involve analysis of the accuracy of the chosen discretisation scheme and sensitivity studies of the optimal solution to the volatility of the electricity prices.

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