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On the Uncertainty caused by the Referendum on Brexit

By

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Abstract

In this paper we investigate "the" uncertainty caused by the 2016 Brexit referendum. In particular, we analyse whether the referendum in itself had a noticeable impact on expectations/behaviour of market participants. To investigate this, we analyse two survey-based indicators and a financial variable, namely the consumer confidence index, the economic policy uncertainty index and the GBP/Euro exchange rate. In the first step we estimate the law of motion of these variables using a state-space model in the time domain. In the second step, we transfer these results into the frequency domain. We find that certain indicators changed very soon after the referendum whilst other indicators reacted to the referendum by changing their medium and long-term behaviour. For those variables it is clear that the short-term reaction to any shock is fairly limited leading to the wrong conclusion that the referendum did not have any impact on them. In fact, the impact will only be seen much later than the original shock. In the opposite case, the wrong conclusion is that the reaction to the referendum is only visible in the short term, but not in the long-run. Therefore, we highlight that the dynamics caused by the referendum are of complex nature which may yet have to materialise. That implies that negative consequences of the referendum alone (never mind the actual Brexit) will only become visible well after the referendum by which time they may not be associated with the referendum anymore. However, we also show that there are short term consequences of the referendum.

Keywords: Brexit Referendum, Volatility, Time-Frequency Analysis, Uncertainty.

JEL Codes: C22, C49, D80, G14

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1. Introduction

On the 23rd of June 2016 the British government held a referendum asking the people to decide on whether or not they wanted to stay in the European Union (EU). The results of the referendum revealed that 52% of the UK population wanted to exit (Brexit) whilst 48% voted to remain in the EU. The political campaign leading to the referendum and its outcome has caused significant division within the UK highlighting highly polarised views along the various regional, intergenerational and socio-economic boundaries. The Brexit vote has considerably increased economic uncertainty in the UK and beyond its geographical borders. Since Article 50 of the Lisbon Treaty was triggered in March 2017, the UK government and EU have been negotiating the terms and conditions of the separation, however very little has been achieved and there still remains a lot of uncertainty around policy, regulations and laws relating to the movement of labour, goods and services, and capital and information. There are also conflicting reports on the actual cost and economic impact of Brexit and many commentators have argued that the main effects of Brexit are difficult to predict due to lack of clarity and vision from both the UK and EU political establishments.

As an immediate impact of UK's decision to exit the EU the pound lost nearly 8% of its value against the dollar, the biggest drop since 1985 and the FTSE 100 index dropped by 3.5% the day after the results of the referendum was announced. (https://www.theguardian.com/business/2016/jun/24/ftse-100-and-sterling-plunge-on-brexit-panic). According to Nikolka and Poutvaara (2016) uncertainty surrounding Brexit vote has already led to the decrease or postponement of private investment worth 65.5 billion pounds.

Many economists and commentators, including the Bank of England, predicted that the immediate consequence of Brexit would be that the UK economy was most likely to slip back into recession, households and businesses would delay spending, unemployment would increase, the pound would fall in value against the dollar and the inflation would increase (https://www.theguardian.com/business/2016/may/12/bank-of-england-keeps-interest-rates-on-hold-as-brexit-fears-bite). However, UK growth did not fall, although it remained below that of the Eurozone and that of the US in 2017.

Reith et al (2016) use different volatility indices such as VStoxx, EuroStoxx 50 and VDax to measure the change in uncertainty as a result of external shocks such as the Brexit vote. Their paper shows that both volatility indices spiked on the morning after the Brexit vote, similar to the spikes at the height of the financial crisis in 2008. Analysis using Vector Autoregressive models show that uncertainty shocks lowers the Euro zone GDP by 0.2% over the next year. The impact of the shock is still felt two years after the initial shock, this is accompanied by a 0.1 % rise in unemployment and 0.7% fall in investment.

The existing literature on uncertainty and economic reviews the reasons why uncertainty effect the economy and the different pathways affected by uncertainty. One pathway looks at the impact of uncertainty on investment and household consumption. The uncertainty regarding the future can have a 'wait and see' effect on businesses and households, who will be more inclined to postpone economic activity until they have a clearer idea of the future, thus having a negative impact on the economy. This is mainly due to the irreversibility of investment projects. Bernanke (1983) focusses on the impact of uncertainty on irreversible investment decisions, where uncertainty creates investment cycles by temporarily increasing the benefits of increased information gained by waiting. This paper uses the concept of 'option value' associated with irreversibility to show that under uncertainty the investor will prefer to postpone his commitment to invest if the improved information is more valuable than short-run return, thus making dynamics of investment sensitive to expectations of uncertainty. Bernanke uses the example of an energy cartel to support this claim and high uncertainty and possibility of new information in the future will pause investment because of the option value of waiting. Pindyck (1991) is consistent with the findings of Bernanke.

Empirical studies reveal that increases in uncertainty raises unemployment and lowers inflation and the short-term nominal interest rate both in US and UK economies (Leduc and Lui, 2016). They measure the impact of macroeconomic effects of uncertainty shocks, by using empirical data on perceived uncertainty by consumers and businesses in VAR models and conclude that uncertainty shocks act like negative aggregate demand shocks as they raise unemployment, reduce inflation at the same time, resulting in a monetary policy response of lowering nominal interest rates. They also find evidence that uncertainty can deepen recessions and hinder recovery. This is consistent with Basu and Bundick (2011) who use calibrated non-competitive, one sector dynamic stochastic DSGE model with sticky process generate simultaneous drops in output, consumption, investment and hours worked as a response to uncertainty shocks. However, they are also able to show in their paper, by introducing price flexibility in the DSGE model uncertainty does not produce business cycle co-movement and results in reduction in consumption but a rise in output, investment and hours worked. Ramey and Ramey (1994) conduct an empirical analysis that demonstrates a strong negative link between volatility/uncertainty and growth. Gilchrist and Williams (2005) show, using a putty-clay model of capital accumulation that a rise in idiosyncratic uncertainty lowers investment at micro-level but raises overall investment at aggregate level.

A second channel investigates the impact of uncertainty when adjustment costs are included. Bloom (2009) offers a structural framework to analyse the uncertainty shocks, creating a model with time-varying second moment of the driving process and including non-convex adjustment costs. He uses firm-level data to estimate the model and uses this parameterised model to simulate large macro uncertainty shocks. The results show that a large temporary uncertainty shock generates a rapid drop, rebound and overshoot in output, employment and productivity growth. The article demonstrates that 4 months after the shock there is a rapid drop in hiring and investment rates because of the option value of waiting. Aggregate productivity also drops in this period as it reduces the level reallocation from low to high productivity firms. However, as the uncertainty has subsided business activity bounces back and there is a rebound in investment and hiring activity. The increased volatility in business conditions leads to a milder long run overshoot in employment, productivity and investment rates. These results are entirely consistent with the empirical studies conducted in this paper using VAR models.

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Caballero and Engle (1999), Caballero, Engel & Haltiwanger (1995) Engel, and Caballero (1994), Abel and Eberly (2001) and Doyle and Whited (2001), Thomas (2002) and Veracierto (2002) all argue that non-convex adjustment costs and irreversibilities play a central role in explaining investment dynamics. Cooper & Haltiwanger (2006) summarise that in the absence of adjustment costs, investment is excessively responsive to shocks. However non-convex and irreversibility models are able to explain inactivity and investment bursts consistent with empirical data.

Bentolila and Bertola (1990), Hamermesh (1989), Caballero &Engel (1993), Davis & Haltiwanger (1992) Caballero & Engel (1993) Caballero & Engel & Haltiwanger (1997) Cooper, Haltiwanger and Willis (2004) look at labour cost adjustments on investment dynamics and uncertainty. Bentolila and Bertola (1990) argue that high firing costs can explain the dynamic behaviour of European employment in the 1970's and 1980's before and after the first oil shock 1973. They demonstrate with help of a stylised model that the oil shock caused firms to stop hiring by reducing employment by attrition rather than mass firing due to high firing costs. Hiring and firing were both negatively impacted by uncertainty causing fall in turnover and stagnant employment levels.

Shapiro (1986), Hall (2004) Merz & Yashiv (2007) carry out joint estimation with labour and investment adjustment costs. Bloom, Bond and Van Reenan (2007) focus on partial equilibrium micro- models and use firm level data and fluctuations in the levels of uncertainty. They find that the impact of uncertainty not only has a negative effect on investment but also reduces the responsiveness of investment to demand shocks. They factor in the labour and capital adjustment costs to understand the dynamics of employment, investment and productivity in response to variations in uncertainty level. There model shows that in the short run investment will respond more cautiously to fiscal or monetary stimulus at higher levels of uncertainty.

The third pathway looks at the impact of volatility on growth. Ramey and Ramey (1994) conduct an empirical analysis that demonstrates a strong negative link between volatility and growth. Through empirical analysis they conclude that there is strong negative impact of volatility of innovations on economic growth. They suggest the variance of innovations to output reflects uncertainty within the economy and can be linked to volatility in government spending. This empirical study supports the finding of the Ramey and Ramey (1991) paper where the cost of volatility is directly linked to firms' decision-making errors under uncertainty. Interestingly Ramey and Ramey (1994) find very little evidence of the negative impact the investment share of GDP in the link between volatility and growth.

Aghion et al (2005) considers how idiosyncratic liquidity risk increases aggregate volatility of investment allocation thus affecting growth negatively. Aghion et al (2005) demonstrate that higher degree of credit control leads to a higher sensitivity of both composition of investment and mean growth to exogenous shock, as well as a stronger negative impact of volatility on growth.

In this paper we follow a different approach to measure uncertainty. We use a Time-Frequency Analysis to decompose the variance of an indicator as done in Hallett and Richter

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(2004, 2006a, 2006b, 2008, 2009a, 2009b, 2011a and 2014) and Correia et al (2016). The Time-Frequency Analysis allows us to extract more detailed information from economic indicators. For example, common (descriptive) analysis usually focusses on changes in trend of say consumer confidence index. A change towards a downward trend indicates a loss in consumer confidence and therefore more uncertainty regarding private consumption expenditure in the future. The Time-Frequency Analysis allows us to detect not only the new trend, but also the volatility surrounding it. In particular, we can measure whether the volatility is actually more important than the trend. We can also measure if and when this importance has changed. In other words, we can differentiate between what we call an "explicit uncertainty" and an "implicit uncertainty". Explicit uncertainty is observable when for example an indicator variable in some way shows a worsening behaviour such as the economic sentiment indicator assumes a lower value due to some event. For the implicit uncertainty we look at the variance of a variable and its importance for the data generating process. If the variance increases and also has a higher importance than say a trend then that indicates a higher uncertainty. It is common practice in finance, for example, to interpret an increase of the standard deviation as an increase of uncertainty. The Time-Frequency Analysis decompose the variance into its cyclical components and then pays particular attention to the short-term behaviour of the variance in particular. The higher is the influence of short-term cycles the higher is the uncertainty. Furthermore, as we know the date of the Brexit referendum, we are able to check whether this implicit uncertainty has changed since the Brexit referendum. To do so, we look at monthly data of some selected variables. We find that implicit uncertainty is only gradually changing, but increasing in relative terms. Explicit uncertainty is visible in almost all variables. What that means is that economic agents are rather certain about the negative consequences of Brexit especially in the long-run. However, there is also an uncertainty regarding the short-term prospect of Brexit.

This paper is structured as follows: The next section will explain our methodology. Section 3 will present the results and section 4 concludes.

2. Methodology

2.1 A Brief Introduction to Time-Frequency Estimation

The Fourier transform (FT) is the standard tool for spectral analysis in the area of signal processing. However, the FT is inadequate when the signal is nonstationary. Classical Fourier techniques only reveal the overall frequency content of these signals. Often it is more important to know which frequency components are present, when they are present, and how they change over time. As conventional representations in the time domain or the frequency domain are evidently insufficient in the situations described above, an obvious solution is to seek a representation of the signal as a two-variable function whose domain is the two-dimensional (t, f) space. Its constant t-cross section shows the frequency or frequencies present at time t, and a constant f-cross section the time or times at which frequency f is important. Such a representation is a **time-frequency representation**.

Spectral analysis decomposes the variance of a sample of data across different frequencies. The power spectrum itself then shows the relative importance of the different cycles in creating movements in that data, and hence describes the cyclical properties of a particular time series. It is assumed that the fluctuations of the underlying data are produced by a large number of elementary cycles of different frequencies. Furthermore, it is usually assumed that the contribution of each cycle is constant throughout the sample. However, as Chauvet and Potter (2001) show for the US, business cycles cannot be assumed to be constant. Hence, the spectrum would not be constant over time due to the changing weights associated with each of the elementary cycles. A "traditional" frequency analysis cannot handle that case. But in recent years a time frequency approach has been developed which can do so. It depends on using a Wigner-Ville distribution for the weights (see for example: Matz and Hlawatsch, 2003). In this paper we use a special case of the Wigner-Ville distribution, namely the "short time Fourier transform" (STFT). The STFT catches structural changes (here interpreted as changes of the underlying lag structure in accordance with Wells, 1996), but assumes local stationarity. However, the STFT has the disadvantage that a window has to be chosen, for which the transformation is done. The choice of this window is crucial for the result. If the window is too large then a structural change may not show up, if the window is to small then not enough data may be available to present all frequencies. Hence, the classical STFT has an arbitrary element in it[2]as it is pointed out by Raihan et al (2005). In this paper, we apply an estimation independent of the window size. The Kalman filter adds a new observation whenever it becomes available and the weight of this new observation is determined by the Kalman Gain. Using the new observation all parameters are estimated again. So, we benefit from the Kalman filter procedure in terms of how it handles structural breaks without having to pre-determine a window. At the same time, as the Kalman filter yields a time series for each parameter we use this time series to calculate the Fourier transform. As a result, we have an STFT with an increasing window.

All the data in this paper (including the Eurozone data) are from Thomson Reuters DataStream. We use monthly data for several indicators from 2012:9 to 2018:3.

2.2 The Model

In order to analyse the implicit uncertainty we estimate an AR(p) process for each variable individually. That is, we estimate the data generating process of each variable separately. In order to allow for the possible changes in the parameters, we employ a time-varying model by applying a Kalman filter to the chosen AR(p) model as follows:

$$y_t = \alpha_{0,t} + \sum_{i=1}^{9} \alpha_{i,t} y_{t-i} + \varepsilon_t \tag{2.1}$$

with

$$\alpha_{i,t} = \alpha_{i,t-1} + \eta_{i,t}, \text{ for i=0...9} \tag{2.2}$$

[2] It should be mentioned though, that wavelets, for example are more flexible than STFT, but crucially depend on the choice of the wavelet function.

and $\varepsilon_t, \eta_{i,t}$ are i.i.d. $(0, \sigma_\varepsilon^2)$ and $(0, \sigma_\eta^2)$, for i= 0…..9, respectively.

In order to run the Kalman filter we need some initial parameter values. The initial parameter values are obtained estimating them by OLS using the entire sample (see also Wells, 1996). Given these starting values, we can then estimate the parameter values using the Kalman filter. We then employed a general to specific approach, eliminating insignificant lags using the strategy specified below. The maximum number of lags was determined by the Akaike Criterion (AIC), and was found to be nine in each case. Each time we ran a new regression we used a new set of initial parameter values. Then, for each regression we applied a set of diagnostic tests shown in the tables in Appendix 2, to confirm the specification found. The final parameter values are filtered estimates, independent of their start values.

Using the above specification implies that we get parameter values for each point in time. Hence, a particular parameter could be significant for all points in time; or at some but not others; or it might never be significant. The parameter changes are at the heart of this paper as they imply a change of the lag structure and a change in the spectral results. We have therefore employed the following testing strategy: if a particular lag was never significant then this lag was dropped from the equation and the model was estimated again. If the AIC criterion was less than before, then that lag was completely excluded. If a parameter was significant for some periods but not others, it was kept in the equation with a parameter value of zero for those periods in which it was insignificant. This strategy minimised the AIC criterion, and leads to a parsimonious specification. Finally, we tested the residuals in each regression thus achieved for auto-correlation and heteroscedasticity.

The specification (2.1) – (2.2) was then *validated* using two different stability tests. Both tests check for the same null hypothesis (in our case a stable AR(9) specification) against differing temporal instabilities. The first is the fluctuations test of Ploberger et al (1989), which detects *discrete* breaks at any point in time in the coefficients of a (possibly dynamic) regression. The second test is due to LaMotte and McWorther (1978), and is designed specifically to detect *random* parameter variation of a specific unit root form (our specification). We found that the random walk hypothesis for the parameters was justified for each country in our sample (results available on request). The possibility of non-stationary parameters does not affect our estimation therefore. Finally, we chose the fluctuations test for detecting structural breaks because the Kalman filter allows structural breaks[3] at any point and the fluctuations test is able to accommodate this.[4] Thus, and in contrast to other tests, the fluctuations test is not restricted to any pre-specified number of breaks.

[3] There are, of course, different definitions of what constitutes a structural break. In this paper, we follow Hansen (2001) who defines a structural break if at least one parameter of the model has changed. Hence, for a structural break to occur it is irrelevant what caused the change of the parameter, as long as the parameter changed. That implies for example, that German unification results in a change of parameter values, which is then interpreted as a structural change.

[4] Note that all our tests of significance, and significant differences in parameters, are being conducted in the time domain, *before* transferring to the frequency domain, because no statistical tests exist for calculated spectra (the transformations are nonlinear and involve complex arithmetic). Stability tests are important here because our spectra are sensitive to changes in the underlying parameters. But with the extensive stability and specification tests conducted, we know there is no reason to switch to another model that fails to pass those tests.

Once this regression is done, it gives us a time-varying AR(p) model. From this AR(p) we can *calculate* the short–time Fourier transform, as originally proposed by Gabor (1946), in order to calculate the time-varying spectrum. We briefly introduce the STFT here: for details, the reader is referred to Boashash (2003). The basic idea is to find the spectrum of a signal x(t), at time t, by analysing a small portion of the signal around that time.

Consider a signal $s(\tau)$ and a real, even window $w(\tau)$, whose Fourier transforms are S(f) and W(f) respectively. To obtain a localised spectrum $s(\tau)$ at time $\tau = t$, we multiply the signal by the window $w(\tau)$ centred at time $\tau = t$. We obtain

$$s_w(t,\tau) = s(\tau)\, w(\tau - t) \tag{2.3}$$

We then calculate the Fourier transform w.r.t. τ which yields

$$F_s^w(t,f) = \underset{\tau \to f}{\mathcal{F}} \left\{ s(\tau)\, w(\tau - t) \right\} \tag{2.4}$$

$F_s^w(t,f)$ is the STFT. It transforms the signal into the frequency domain across time. It is therefore a function of both. Using a bilinear kernel and a Gabor transform (the time series is stationary, but may contain parameter changes), Boashash and Reilly (1992) show that the STFT can always be expressed as a time-varying discrete fast-Fourier transform calculated for each point in time. That has the convenient property that the "traditional" formulae for the coherence or the gain are still valid, but have to be recalculated at each point in time. The time -varying spectrum of the growth rate series can therefore be calculated as (see also: Lin, 1997):

$$P_t(\omega) = \frac{\sigma^2}{\left| 1 + \sum_{i=1}^{9} \alpha_{i,t} \exp(-j\omega i) \right|_t^2} \tag{2.5}$$

where ω is angular frequency and j is a complex number. The main advantage of this method is that, at any point in time, a power spectrum can be calculated instantaneously from the updated parameters of the model (see also Lin, 1997). Similarly, the power spectrum for any particular time interval can be calculated by averaging the filter parameters over that interval. This would then result in the "traditional" spectra.

3. Empirical Results

3.1 Economic Policy Uncertainty Indicator

The first uncertainty variable we look at is the economic policy uncertainty (EPU) as measured by Scott et al. (2016). The European policy-related economic uncertainty index is an index based on newspaper articles regarding policy uncertainty. For each country, two newspapers are selected. For the UK they are The Times of London and Financial Times. For the EPU the number of newspaper articles containing the terms uncertain or uncertainty,

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economic or economy, and one or more policy-relevant terms is counted. The EPU is normalised to a mean of 100. We collected monthly data for the EPU form 2001:1 to 2018:3. The following figure 1 shows the development of the EPU over time for the UK.

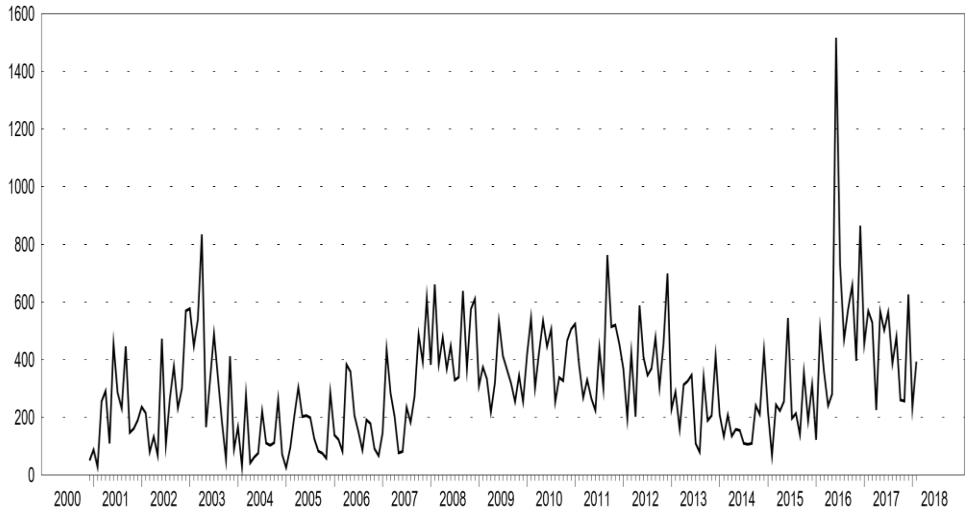

Figure 1: EPU

As can be seen from Figure 1, the EPU shows a spike in the month of the referendum. This increase in volatility, it seems, was reduced in the following months and it seems, that the indicator moved back to its "normal" behaviour. Prior to the referendum the standard error, which is interpreted as an indicator of uncertainty, the EPU was 186.31 in comparison to 140.52 for the period after the referendum. Hence, looking at the standard error one could conclude that uncertainty after the referendum was actually less than before. In order to test this inference, we performed a spectral decomposition of the variance. As described above we estimated the data generating process using an AR(p) model. We then transferred the time domain results into the frequency domain. The Kalman filter results are presented in table 1 below:

VAR/System - Estimation by Kalman Filter			
Dependent Variable	DLHU_GDP	Monthly Data From	2001:12 To 2018:03
Usable Observations	196	Std Error of Dependent Variable	164.8481776
R^2	0.99997	Standard Error of Estimate	289.438
Mean of Dependent Variable	290.4109	Sum of Squared Residuals	14241642
Akaike (AIC) Criterion	304.58846	Ljung-Box Test: Q*(28)	25.136
Variable	Coeff	Std Error	T-Stat
Constant	548.8258	236.1231	2.3243

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ECOUNC{1}	-0.78047	0.1832	-4.2598
ECOUNC{2}	-0.29121	0.2006	-1.4518
ECOUNC{3}	-0.13879	0.0360	-3.8543
ECOUNC{7}	0.131698	0.107689	1.2229

Table 1: Regression Results for EPU

The reason why the 2nd and 7th lag of EPU are included in the regression is that at earlier points in time both lags were significant. As described in the previous section we used the model in table 1 to calculate the time-varying spectrum for EPU which is shown in Figure 2.

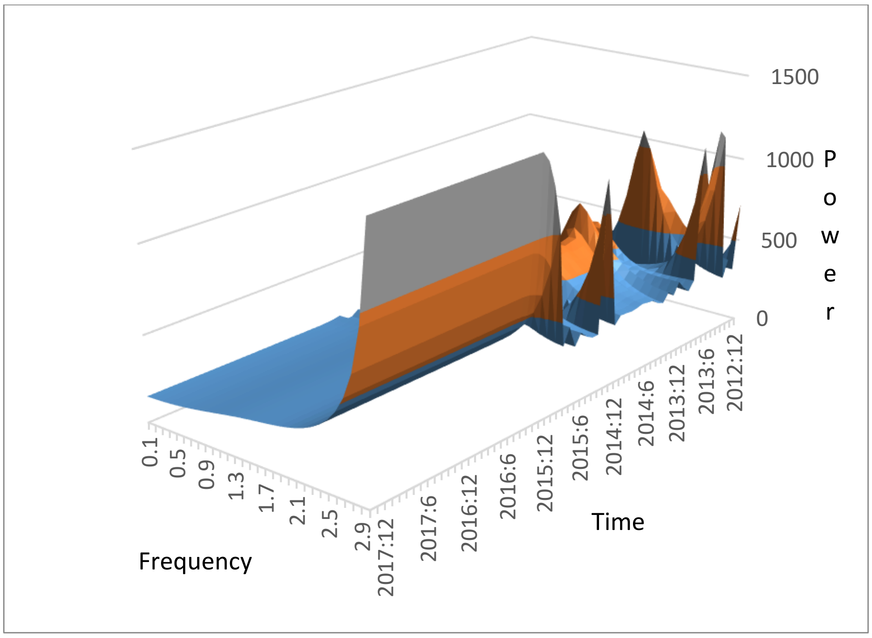

Figure 2: Spectrum of EPU

As can be seen from Figure 2, after the referendum took place most of the variance of EPU is caused by very short cycles which reflects short term uncertainty. So, in difference to the time series properties digital filtering shows that the Brexit referendum had a profound effect on economic policy uncertainty, which actually did not diminish after the referendum but stayed at very high levels. Therefore, EPU does not show an explicit uncertainty, but an implicit uncertainty as reflected by the overwhelming dominance of short-term cycles. We can interpret this implicit uncertainty as the uncertainty where people do not know what to expect and therefore adjust their expectations with the availability of new information on a short-term basis. This is in difference to explicit uncertainty where the uncertainty is expressed in long term-term developments. In the latter case, people would expect something negative to happen, but would not know exactly what. In other words, explicit uncertainty represents the "known unknowns" and implicit uncertainty represents the "unknown unknowns".

3.2 Consumer Confidence Indicator

The UK Consumer Confidence Indicator is conducted by GfK on behalf of the EU, with similar surveys being conducted in each European country. It is conducted among a sample of 2,001 individuals aged 16+ on behalf of the European Commission. Quotas are imposed on age, sex, region and social class to ensure the final sample is representative of the UK population.

We use monthly data from 1974:1 to 2018:3 as provided by Thomson-Reuters. The data is not seasonally adjusted as can be seen from Figure 3.

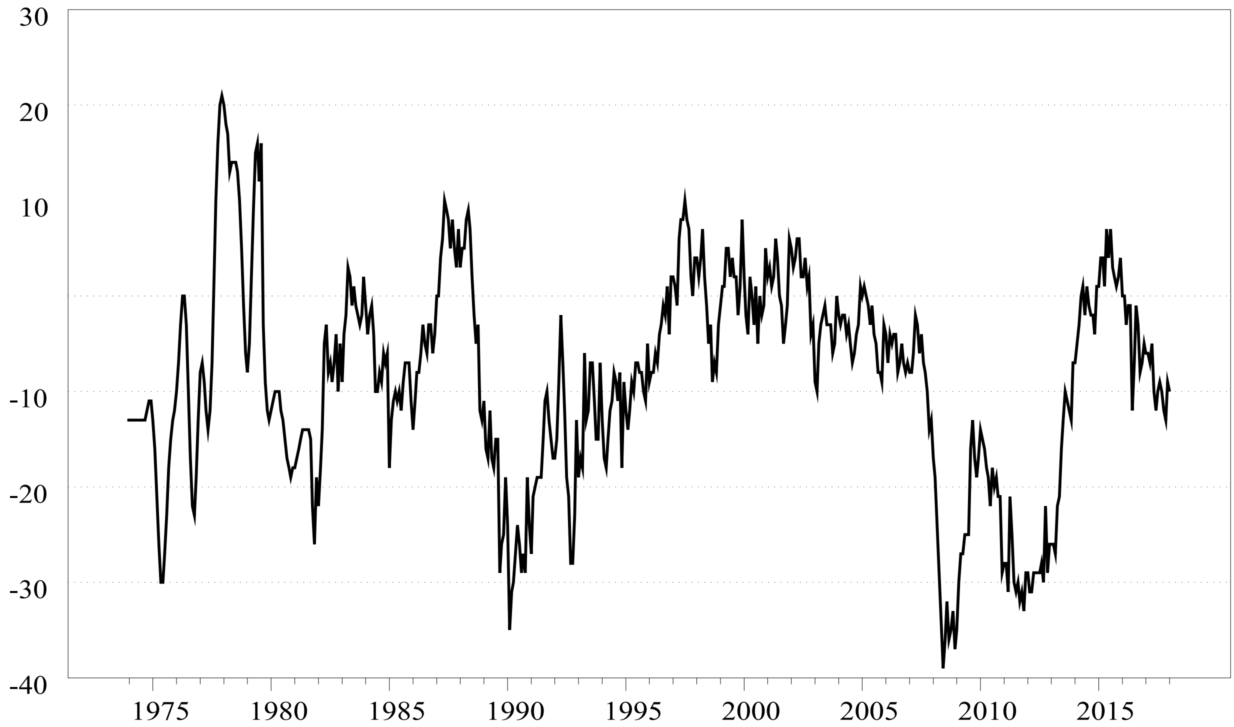

Figure 3: Consumer Confidence Indicator

As can be seen from Figure 3, the consumer confidence index shows a volatile behaviour. However, this volatility is not constant. Depending on the sort of crisis (or not) the volatility is sometimes higher and sometimes smaller. Figure 3 shows that the behaviour of the consumer confidence index can be characterised by a wave with fluctuations around it. The cycle length is not constant. Arguably the longest cycle was from 1990 to 2007. Since then the cycles have shortened. Of course, 2007 represents the financial crisis where one would expect the consumer confidence index to be on a low. When it then increased in 2009/10 it was interpreted as a reversal from the crisis. The Brexit referendum in 2016 clearly shows a new downward trend, which has yet to come to a halt.

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The consumer confidence indicator is an example where we can see some sort of “normal” volatility which then changes in times of crisis. This change of volatility is what we would like to measure and in particular its importance in comparison to all other volatilities.

In order to do so, we estimated the following the state space model:

VAR/System - Estimation by Kalman Filter			
Dependent Variable	CONCON	Monthly Data From	1975:01 To 2018:03
Usable Observations	519	Std Error of Dependent Variable	11.4935
R^2	0.99937	Standard Error of Estimate	4.5107
Mean of Dependent Variable	-8.6795	Sum of Squared Residuals	10030.8239
Akaike (AIC) Criterion	4.5808	Ljung-Box Test: Q*(28)	55.699
Variable	Coeff	Std Error	T-Stat
Constant	0.0791	0.0171	4.6322
CONCON{1}	0.6403	0.0765	8.3658
CONCON{5}	0.3171	0.6817	0.4652
CONCON{7}	0.0594	0.3845	0.1545

Table 2: Regression Results for Consumer Confidence Index

Table 2 shows that the estimated model for consumer confidence is a AR(7) model, where at the end of the sample the last two lags are insignificant. However, these lags were significant earlier which is why they were kept in the sample.

From these results we calculated the appropriate time-varying power spectrum.

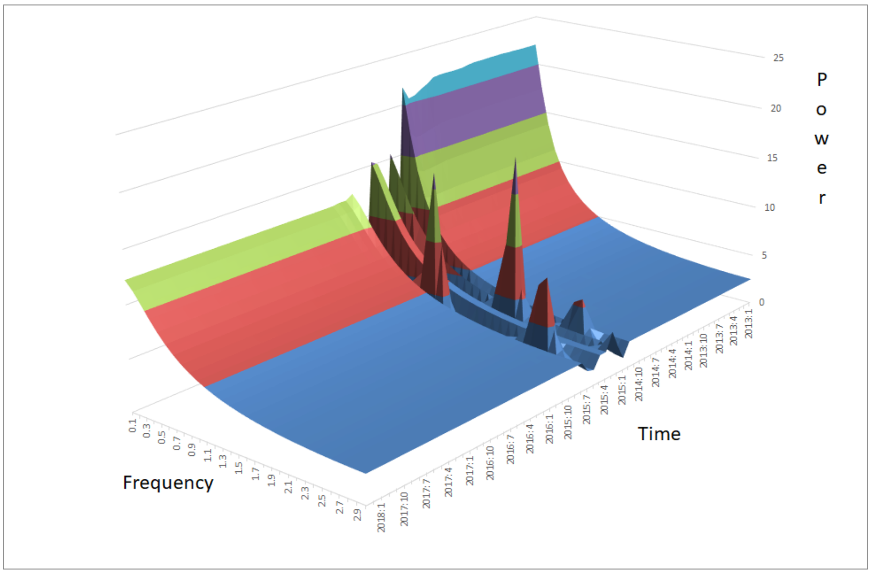

Figure 4: Time-Varying Power Spectrum of Consumer Confidence

Figure 4 shows that throughout the sample, the long-run trend is the most important cycle. However, as smooth as the diagram appears to be, in period of the referendum and afterwards, this smoothness was disturbed as visible in the short-term spikes and waves. Once this shock was digested the system returned to a new steady state. As before, the most important cycle is the long-run trend. Clearly, the long-run trend lost power due to the Brexit referendum. All other cycles remained in terms of their influence. Therefore, the result of the referendum certainty is reduced in terms of the loss of power on the trend. Hence, relatively speaking uncertainty increased. This change of the implicit uncertainty is only revealed in the digital resolution of the volatility. Therefore, we observe a higher volatility as a result from the loss of the long-run certainty in contrast to a higher short-run volatility.

3.3 The Exchange Rate

The next variable we are analysing is the British Pound/Euro exchange rate. The exchange rate was collected as monthly data from Thomson/Reuters DataStream and covers the period from 1975M1 to 2018M4.

Over the sample the exchange rate has a mean of 0.71GBP/Euro and a standard deviation of 0.09. The minimum is 0.52GBP/Eur in 1981 and the maximum is 0.96 which was reached at the peak of the financial crisis in 2007. Figure 5 shows that the value of the British currency was higher at the beginning at the sample than at the end. It seems that with the exception of the early 2000s there is an upward trend in the time series. The monthly data also shows volatility of the exchange rate, which varies depending on the particular event it caused it. Obviously, volatility in the financial crisis 2007 was higher than in the years before that.

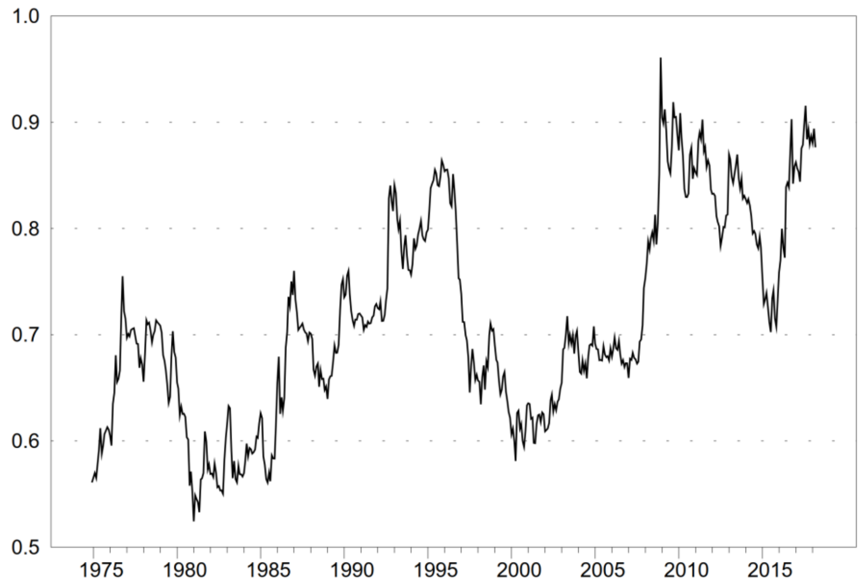

Figure 5: GBP/Euro Exchange Rate

After the financial crisis in 2007 the British Pound enjoyed a period of appreciation, which clearly came to a halt with the Brexit referendum. From 2016 to 2018 the Pound lost nearly 20p of its value to the Euro. Whilst this does not reflect the largest depreciation in the sample period it is nevertheless an event that is clearly visible in the graph and reflects the explicit volatility in the market.

As before we used an AR(p) model, where p =3, to analyse the data generating process of the exchange rate. The following table shows the Kalman filter results.

VAR/System - Estimation by Kalman Filter			
Dependent Variable	GBPEUR	Monthly Data From	1975:01 To 2018:03
Usable Observations	519	Std Error of Dependent Variable	0.0954
R^2	0.9960	Standard Error of Estimate	0.1038
Mean of Dependent Variable	0.7139	Sum of Squared Residuals	5.5540
Akaike (AIC) Criterion	0.1054	Ljung-Box Test: Q*(45)	61.466
Variable	Coeff	Std Error	T-Stat
Constant	0.4441	0.0392	11.3301
GBPEUR{1}	0.3729	0.0542	6.8834
GBPEUR{2}	-0.1071	0.0086	-12.4658
GBPEUR{3}	0.2406	0.0420	5.7310

Table 3: Regression Results for the Exchange Rate

In difference to the consumer confidence model, the memory of the model is fairly short. It only goes back three months, whilst in the previous case it was 7 months. Like before the residuals are not autocorrelated and not heteroscedastic.

The following figure 6 shows the power spectrum of the exchange rate based on above AR(3) model. The power spectrum seems to be fairly stable. By far the biggest power has the long run trend. In difference to the consumer confidence index, this did not change even after the referendum. The power of the long-run trend actually increased after the referendum, which is perhaps somewhat unexpected. It is also remarkable that the short-run volatility is constant throughout the sample.

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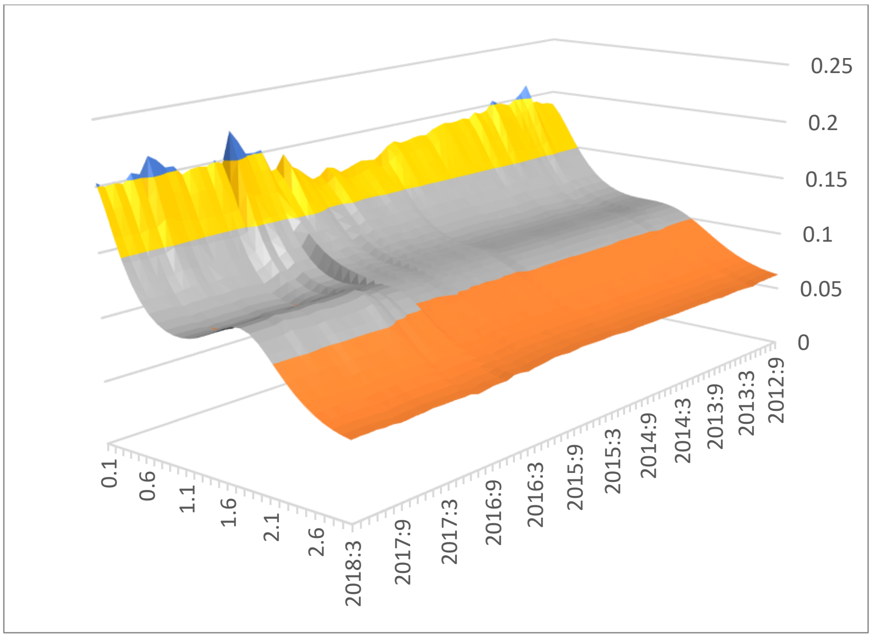

Figure 6: Power Spectrum of the Euro-GBP Exchange Rate

However, the effect of the referendum in itself is clearly visible. In June 2016, volatility increased for all frequencies. But this increase lasted only for three months. Having said that, the referendum did actually significantly change the power spectrum. The cycle at a frequency of 1.7 emerged (previously it as the cycle at 1.5). This new cycle is also stronger than the previous cycle. The frequency of 1.7 corresponds to 3.7 months. The emergence of a new cycle implies higher volatility in the market. As this cycle's power has not changed since its occurrence, it implies that foreign exchange markets have indeed become more volatile since the referendum.

In difference to the consumer confidence index and the economic policy uncertainty index, the increase of uncertainty is reflected here by the emergence of a new medium cycle. Long-run trends still play a dominant role, but short-term uncertainty has not increased (which is not to say that it does not matter).

4. Conclusion

In this paper, we analysed the uncertainty caused by the Brexit referendum in June 2016. We investigated three different indicators: the economic policy uncertainty indicator, consumer confidence indicator and the GBP/Euro exchange rate. Whilst this list is not comprehensive of all potential indicators available our aim was to analyse whether uncertainty has increased and if so, how this increased uncertainty would express itself. In order to analyse uncertainty, we used a time-frequency analysis.

We found that for the three indicators uncertainty showed in different forms. From common finance literature increased uncertainty is usually reflected in a higher volatility. We call this the explicit uncertainty as this uncertainty can be inferred from the time series directly. The implicit uncertainty is visible via the fast Fourier Transform. What we found there is that in one instance increased uncertainty is indeed expressed as an increase in short-term volatility.

However, we also found that increased uncertainty can be expressed in different ways: in case of the consumer confidence index, the increase in uncertainty was visible as a reduction of the long-run certainty. Therefore, relative uncertainty increased as relatively speaking, short-term uncertainty gained relative weight. In the last case, the increased uncertainty led to the emergence of a new medium cycle in addition to the previous dominant cycle(s).

The relevance of these results is that these results help to explain, why since the referendum the economic situation in the UK has not changed as dramatically as it was anticipated by some economists. The referendum had obviously an effect on the volatility of different variables. But due to the precise changes of the volatility some variables will show in the short-term economic effects (economic policy uncertainty), whilst for other variables the effect will only be visible in the medium or long-run. Therefore, some "indicators" would not show an immediate change of their volatility which does not mean that there is no change. This effect, makes forecasts in general more complex. Especially for the last two cases, generally, long-run and medium-term forecasts are more unreliable than short-run forecasts. Even if these forecasts are accurate, the effects of the higher uncertainty may only be felt some time after the shock occurred by which time some other shocks may have happened as well. These new shocks may well supersede the original shock. As a result, policy makers may be inclined to conclude financial markets do not show as much uncertainty as "expected" leading to wrong conclusions regarding financial markets' stability for example.

Therefore, what this paper shows is that even the referendum in itself which did not immediately change the economic framework (yet) had already an impact on the volatility of certain variables and by doing so changed the underlying data generating process. When the UK leaves the EU, we would expect this volatility to increase even further with all the negative implications associated with that. Of course, this paper only investigates a limited number of indicators and future research should cover more variables. Also, if and when Brexit happens that will change the dynamic behaviour of economic variables, which is well worthwhile to be investigated.

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