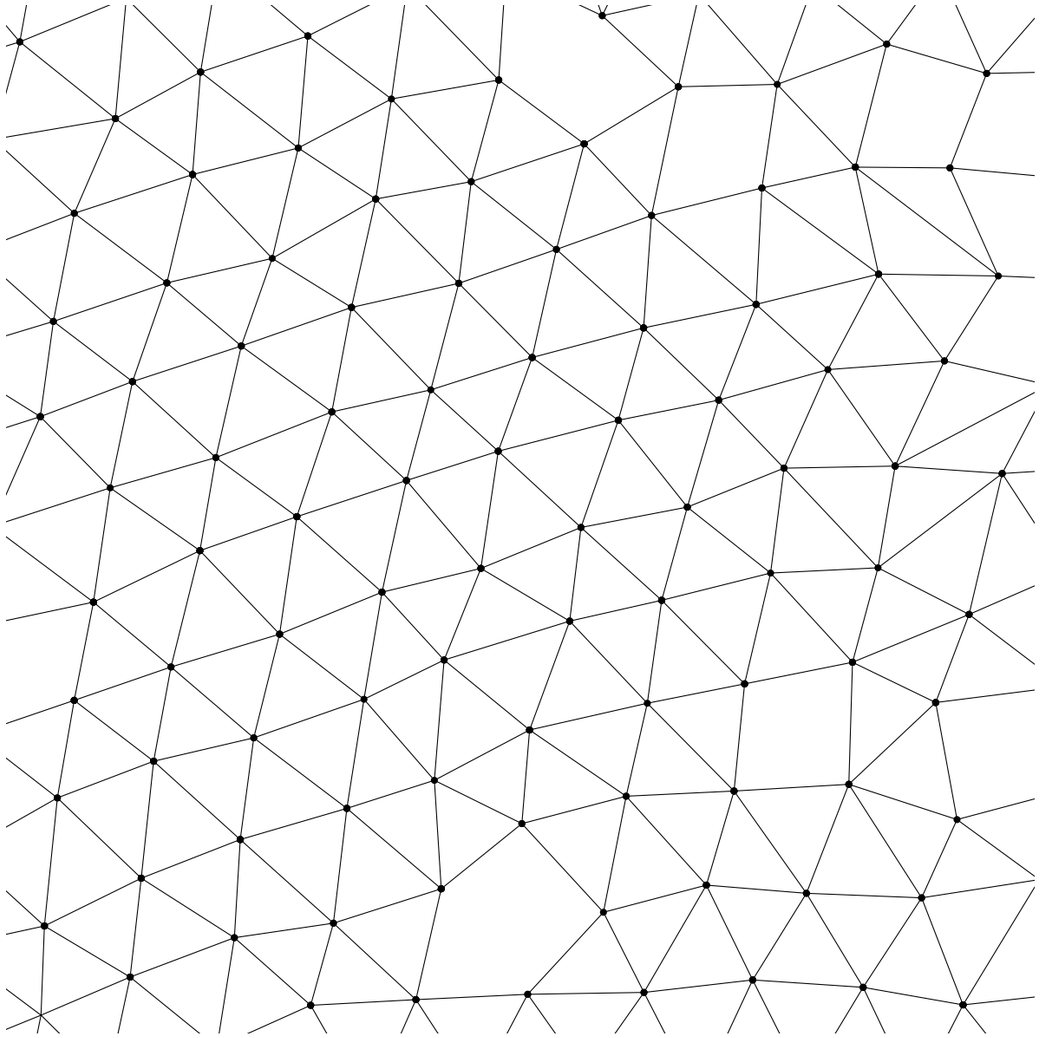
chapter one

Rethinking representation



The introduction sets out the initial position of text as design representation. Fundamentally the proposition is that Chomsky's dictum – that finite syntax and lexicon can nevertheless generate an infinite number of useful (well-formed) structures – can be applied to artificial languages, and that texts can be written in those languages to generate architectural objects, taken to mean 'well-formed' configurations of space and form. This is the generative algorithm and the idea is that a generative algorithm is a description of the object just as much as the measurement and analysis of the object, the illustration of the object and the fact of its embodiment in the world.

The position here is that the text we are looking at, being an artificial language, usually depends for its embodiment on some hardware – the engineering product of the Turing machine – and this hardware affords some species of representation, from simple graphics all the way up to programmable hardware, 3D printing and immersive virtual worlds. But this aspect is simply an unfolding of the underlying algorithm, which is still the original representation. It would be possible to orchestrate 300 human beings to obey instructions and so act out the algorithm (like synchronised swimmers) such as in the following.

Some simple texts

As a very first shot, take the example of representing some simple geometric shapes and volumes like the circle, the spheroid and other 3D polyhedra, not using geometry, but small programs written in a dialect of Logo (a venerable Artificial Intelligence (AI) language defined by Seymour Papert, whose history is elaborated in the next section).

Triangles and circles
For the 2D case, this can be verified with a simple experiment using a program with a large number of points in 2D space, initially randomly sprinkled over the plane.

Give each point a rule:

> 'Search through all the other points and find the nearest one to yourself.'
> 'Then move away from this nearest point.'

All the points do this simultaneously.

Of course the problem is that, in backing away from your nearest neighbour, you may inadvertently come too close to someone else, but that is ok because then you just turn around and back away from them. Remember that everybody is doing this at the same time.

To demonstrate how this works we can teach these rules to a computer using the NetLogo language which provides a mechanism for setting up parallel computations very simply. The points are described using 'turtles' – little autonomous computer programs, all of whom obey the program set out below:

```
to repel
ask turtles
[
  set closest-turtle min-one-of other
      turtles [distance myself]
  set heading towards closest-turtle
  back 1
]
end
```

To understand this piece of code, first notice that the whole thing is wrapped up in the clause:

```
to repel

  do something

end
```

This is because we are defining how *to do something* for the computer, so here we are setting out how *to repel*. The stuff between the word 'to' and the word 'end' is the actual code. Then comes the phrase '*ask turtles*'. Who, you might ask, is doing this asking? The turtles are the points in space, they are really a lot of tiny abstract computers, and the global overall observer is, in this statement, sending out a message to all the turtles to run the program enclosed in the square brackets [ ], which is the three sentences:

```
1)  set closest-turtle min-one-of other
    turtles [distance myself]
2)  set heading towards closest-turtle
3)  back 1
```

The turtles are being told:

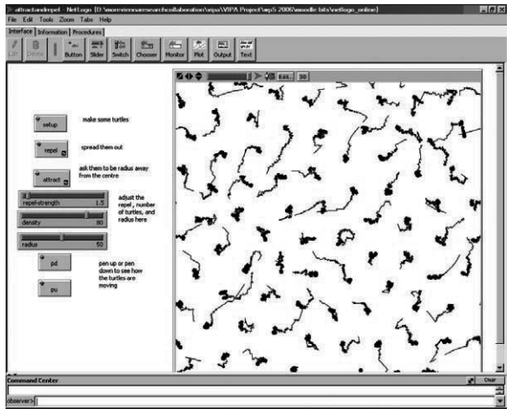
> 'Dear turtles, I would like to ask you to look through all the other turtles to find the one whose distance away is at a minimum.'

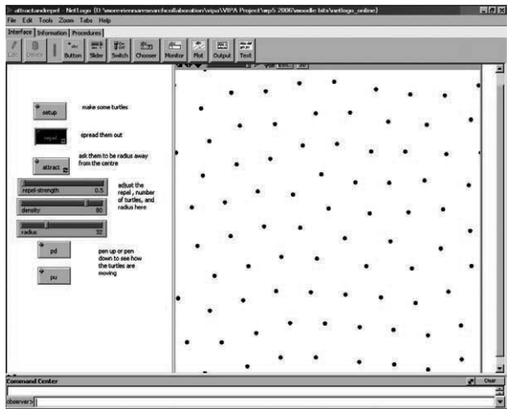
Then they must remember which turtle this is by storing its reference in the name '*closest-turtle*'.

Now the turtles are told:

> 'Set your heading so that you are pointing towards this "closest-turtle", and back off one step.'

Interestingly we also have to tell the computer to address the 'other' turtles as in the human language description . If we just asked all the turtles this would include myself (the one doing the ASKing), and we would get a value of zero and try to walk away from ourselves – not a good idea. This is a good example (the first of many) of how we have to SPELL IT OUT for these supremely pedantic machines.

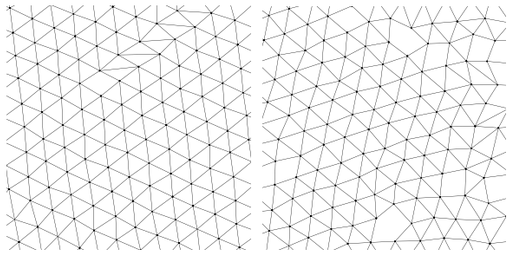




In the top left-hand image, the trails of the turtles are shown moving from the initial random sprinkling to the triangular grid. It takes about 500 steps for the system to settle down, and it can be observed that the turtles quite quickly find a suitable position and then stay there (the trails do not stretch very far, and rarely cross).

These and many other examples of programming in the book are based on NetLogo. This language is a descendant of StarLogo which, in turn, was a parallel implementation of Logo (described in the next chapter), which itself was a development of LISP (see Chapter 3). See Resnic (1994) for a good introduction.

The turtles settle down to a triangular least-effort configuration. See below where the points are linked to show the triangular grid.



To the left, two versions of the outcome running with links are shown. None of these patterns lasts for long; like all dynamic systems the moment can be captured, but is gone and lost for ever by the ceaseless jiggling of the turtles.

Emergent tessellations

With a suitable repel strength, the points all settle down in a triangular pattern because whenever they diverge from this grid they are in an unstable situation and will always fall back into the triangular lattice. The point to note is that these wiggles are not in the algorithm (all it states is the backing off principle outlined above). What would one expect from such an algorithm? At first sight perhaps just aimless wandering; however, it does in fact settle down as if pulled into alignment by some 'force' not implied by the two lines of code. This is an example of 'emergence' – the idea that the program, by operating continuously in parallel, engenders a higher order observation, which could be characterised as a simple demonstration of the principle that the triangular lattice is the least cost-minimum energy equilibrium point for a 2D tessellation, with each point equidistant to six others. Here also is our first example of an algorithm which possessed epistemic independence of the model (in this case the code of the repel algorithm) from the structural output running the algorithm. In other words the stable triangular tessellation (the structural output of the program) is not explicitly written in the rules; which is an example of distributed representation.

Distributed representation

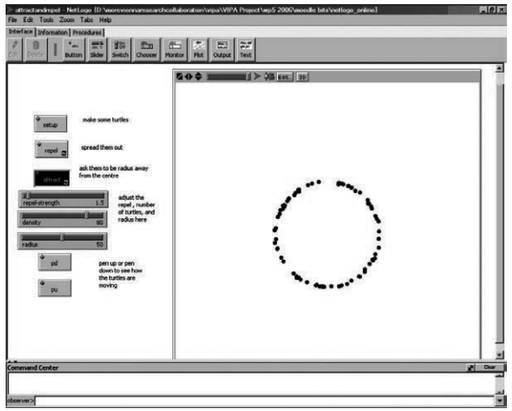
This is also the first example of many that illustrates the notion of distributed representation. The way the algorithm works is to embed the rules to be *simultaneously followed* in EACH turtle. Each turtle (small autonomous computational entity) is running the little program described above with its own decision making – who is nearest to MYSELF – and behaves independently of the other little computers – I turn THIS WAY and back off. The repel algorithm is the only available description we can find in this system, everything else is just general scheduling events and general start stop for the whole simulation, and this representation is present in EVERY turtle. The turtles can interact with each other and have some limited observational powers, for instance they can 'feel' the nearest turtle and take appropriate action, but they do not know about the triangular tessellation since that can only be observed by the global observer – in this case, the person (you) running the simulation on your computer. This distinction between different levels of observer is a key aspect of distributed representation, and will crop up many times in the following pages. It is vital, with distributed representation models, that there is some feedback present between these little autonomous programs; if each one took no notice of its neighbours then nothing would happen. This is evident in the cellular automata shown next and the canonical 'pondslime algorithm' introduced at the end of this chapter.

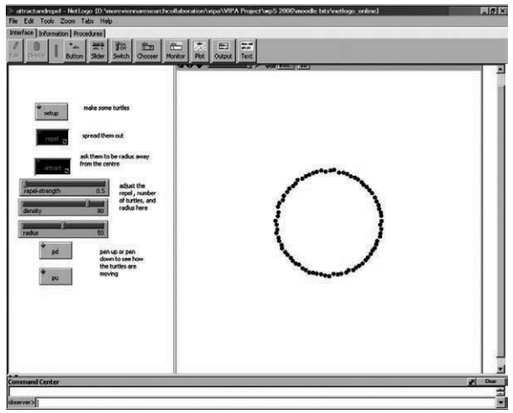
It is instructive to compare this bottom-up small program with the conventional recipe for a triangular tessellation. Of course there are many ways of describing how to draw such a pattern by using a simple wallpaper approach.

Wallpaper algorithm

```
Set out a line of dots at a spacing of 1.
Duplicate this line with an offset of 0.5
   in the x direction and the square root
   of 0.75 in the y direction.
Do this as many times as you like.
```

The square root of 0.75 is the height of an equilateral triangle of side 1 derived from Pythagoras (where height^2 + 0.5^2 = 1^2; so h = √1-0.25), which evaluates to approximately 0.8660254037844386467637231 7075294. This is not a very attractive number and seems to suggest that this algorithm is not capturing the real description of the underlying dynamics, but just mechanically constructing a top-down and rather clumsy measurement of the outcome. This distinction should be remembered when simulations and modelling are discussed elsewhere, as it forms part of the argument in favour of the 'short description' encoded in the generative rule rather than the 'long description' involved in traditional geometry.





IFELSE is an example of one of the key concepts of any programming language: the ability to get the computer to ask a question about which there are a number of things to process. Known as a 'conditional statement', it has many forms, but in this language, in this situation, we use the phrase 'ifelse'.

This construct example has to decide which of two possible routes to take in the flow of the program.

Cheesy illustration: If standing at a fork in the road, with the possiblity of going left or right, you need some way of evaluating the choices open to you. So there you are, what do you do? It happens you have a note from your aunt in your pocket, you take it out and it says:

> {'when reaching a fork in the road, if it's after lunch turn left, else turn right'}

It is clearly just after lunch, so you take the left turn. Problem resolved. (The left turn takes you to the tea rooms, obviously.)

In the script of `attract` the note from your aunt is asking 'if your distance to the centre is less than radius, then take a step back, otherwise step forwards'.

The general notion of IFELSE is that you ask a question, then on the basis of the TRUTH or otherwise of the statement, you choose between two possibilities:

IF <something is true> ***THEN*** DOTHIS
ELSE DOTHAT

that is why it is called IFELSE:

formally
ifelse (conditional expression)
[thing to do if true]
[thing to do if false]

Extending the model – drawing circles with turtles
The following examples are based on the Papert paradigm of allowing the geometry to emerge from the algorithm rather than being imposed from outside. In this case the geometry is based on the circle, which is then extended to cover more complex geometries such as the voronoi (emergent tessellation). These are 'illustrations of consensus' because the bit you can see (the two images on the page opposite) is the emergent result of all the components of the system (turtles mostly) finally reaching some agreement about where to be. The phrase begs the question as to what the turtles are being asked to agree about, and what architectural idea might be involved. Generally, the task is to distribute themselves with respect to two conflicting pressures – that of the group based on some higher order pattern, and that of the individual.
Papert points out that the equations:

```
Xcirc = originX + Radius cos (angle)
Ycirc = originY + Radius sin (angle)
```

do not capture any useful information about circles, whereas we can write a small program in NetLogo to get one turtle to walk in a circle by telling it to go forward and left a bit (see Chapter 2 for background on Seymor Papert). The program:

```
To circle
  Repeat 36
  Forward 1
  Turn Left 10
End repeat
End circle
```

requires only English and a familiarity with walking.

As Resnick points out in *Turtles, Termites and Traffic Jams* (1994), with parallel computation we can propose another implementation of the circle using not just one turtle, but many of them. The algorithm is based on the characterisation of a circle as being:

> An array of points all at the same distance from another common point

To do this with turtles we:

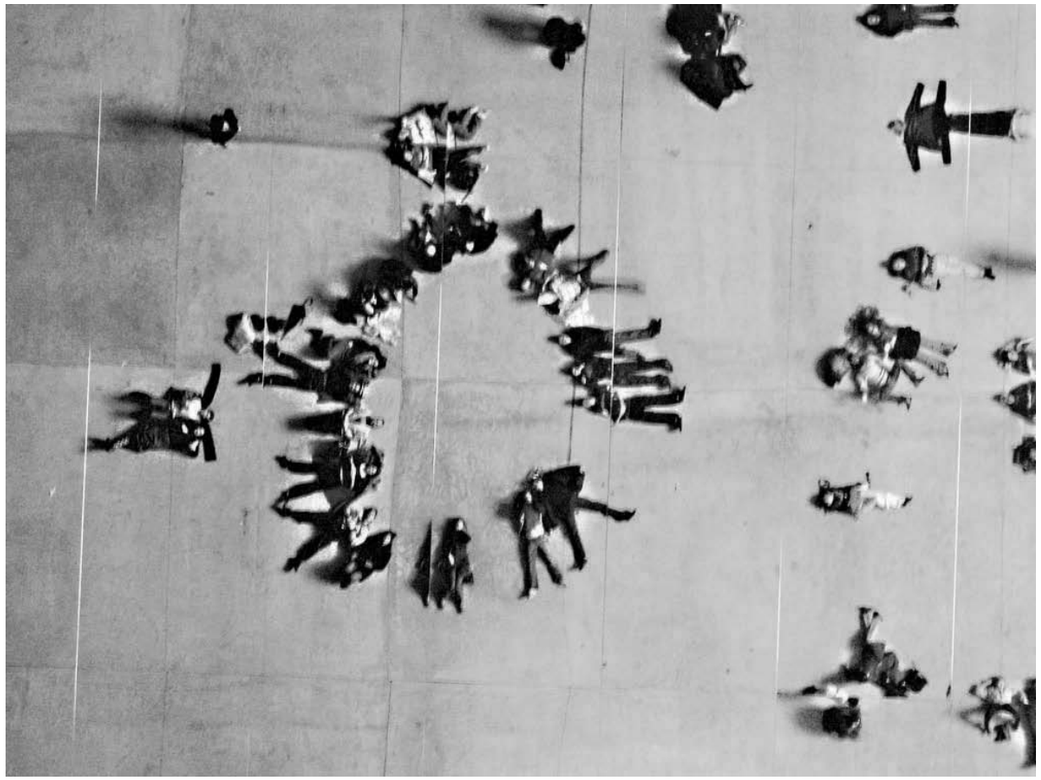
- create a lot of turtles at random;
- get each turtle to turn towards the centre of the circle;
- get each turtle to measure the distance between itself and this centre point;
- if this distance is less than the desired radius, then take a step back (because you are too near);
- if it is greater, then take a step forward (because you are too far away); and
- go on doing this for ever.

This procedure can be written in NetLogo as:

```
to attract
ask turtles
[
  set heading towardsxy 0 0
  ifelse ((distancexy 0 0 ) < radius)
       [bk 1]
       [fd 1]
]
end
```

Notice that nowhere in the procedure is it given where the turtles are to walk to, they just walk back and forth. In fact the 'circle' is only apparent to the human observer, and while we look at it, it shimmers into being rather than being constructed carefully. The result is a ring of turtles defining a circle. In fact there is one more thing to do because just using this process will result in an uneven circle with gaps in as the turtles start off randomly and gather in random spacings around the circumference. How can we get the turtles to spread themselves out? The answer is to do the repel procedure we have already looked at. This version backs off not 1 unit, but a variable amount controlled by a 'slider' on the interface:

```
to repel
ask turtles
[
  set closest-turtle min-one-of other
       turtles [distance myself]
  set heading towards closest-turtle
  bk repel-strength
]
end
```



Illustrations of consensus
A photograph taken while lying on the floor of the Turbine Hall Gallery at the Tate Modern, London, looking up to the mirrored ceiling. It shows how people have arranged themselves in a circular pattern (there is another one forming to the right of the image) without there being any formal 'directive'. The actual geometry is not obvious while walking about the gallery, and only shows up once you lie down on your back and get the God's eye view – when one becomes the external observer. (Thanks to MSc student Stefan Krakhofer for the photograph.)

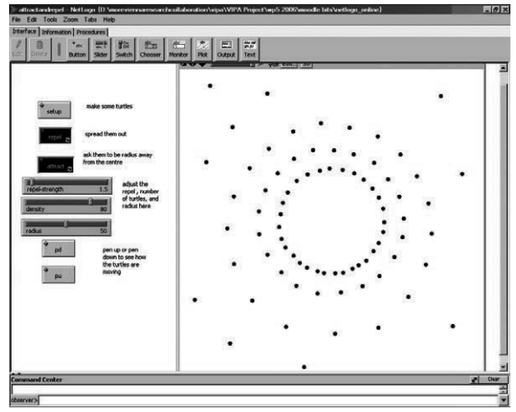
These two procedures use two references to globally define values which affect the system being simulated, called 'radius' and 'repel strength'. These named values are referred to as variables (because they can contain numbers that vary). In NetLogo you can set the variables through the user interface by using sliders.

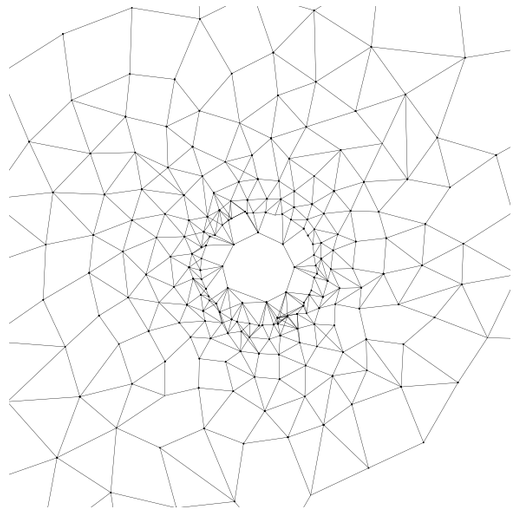
You might say that this is not a 'real' circle, but just a messy thing that is a bit circular. But, like the triangular tesselation example, the classical definition of pi as the ratio of the circumference divided by the diameter is famously unresolvable. In fact, the expansion of pi can be used as the basis for generating a random sequence, as it is impossible to predict the next number in the sequence by any means other than continuing to iterate the division sum. In other words, in our universe circles cannot be identified with whole numbers, every measurement of a circular thing is inevitably a compromise, only resolved by its eventual instantiation into an array of bricks, pieces of steel, etc. So repel and attract (which only use simple additions and no funny ratios) seem more fundamental descriptions, generating the funny ratios out of the process rather than squashing them in by force.

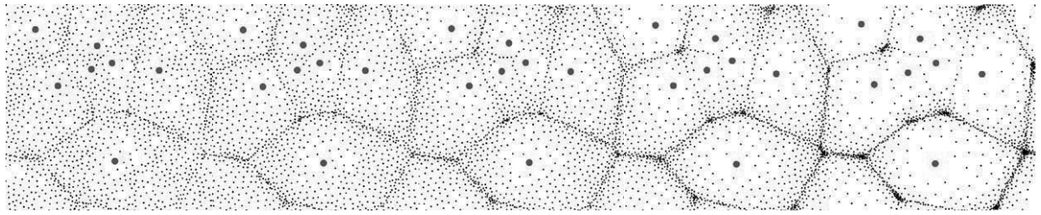
These two variables, 'repel' and 'attract' form a useful test bed for experiments. There is a relationship between the values of the variables such that, if you make the radius very small, then you of course make a smaller circle. If you make the repel strength quite large, then, depending on the number of turtles (another variable), the turtles will find it impossible for all of them to comfortably fit on the circumference. The actual result is quite surprising, as it leads to a series of well-formed rings of turtles at ever-increasing distances from the nucleus. In many ways this could be seen as an example of a Bohr's model of the atom, since the radius is the overall energy of the atom and the repulsion force is the energy level of an electron. (This is intended only as an illustration of the possible explanatory power of these simple models and not a claim to deep physical truth!)

What is undeniable is that, instead of a general fuzzy ring of turtles from the radius outwards, they only inhabit particular rings, which again is not in the model. The text of the algorithm does not include an explicit reference to annular ringyness, but only one circle.

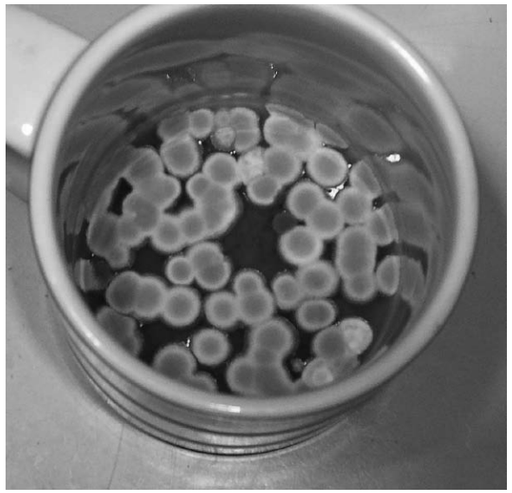
Given the high level of abstraction, we can begin to model more complex shapes and spatial organisations than individual geometric objects without having to do much extra coding, as in the following illustrations. The latter image simply has an additional rule to draw a line between each turtle and its nearest neighbours; see below





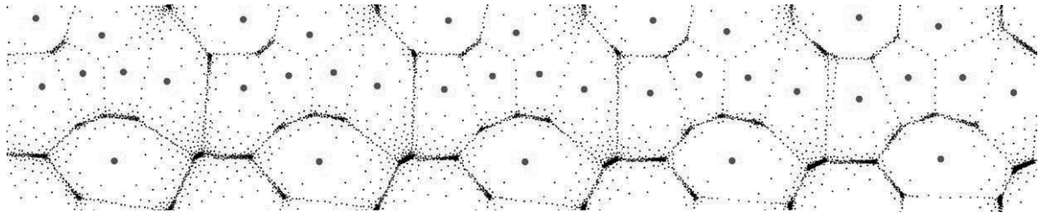


The simulation begins with the two kinds of turtle – 'normal' turtles (little) and 'target' turtles (big) – sprinkled randomly about. Slowly the smaller, normal turtles retreat to the given radius distance in the attract procedure, gathering on the boundaries in ever greater numbers. They cannot go near other targets, but end up in a position which is as far away as possible from all the nearest targets.



If the program models the process to be represented, rather than the graphics of the outcome, it is likely to be a better, shorter model.

This image of mould growing in a coffee cup shows an agglomeration of disc-like elements into a Voronoi like mat.



Extending the model – drawing bubbles

A more complex outcome that we can achieve with only small modifications is the emergent Voronoi diagram (dirichelet tessellations). Voronoi diagrams are conventionally calculated using computational geometry. A Voronoi diagram is a pattern which describes the minimal energy pathways between a set of points. Looking at such a diagram we can see that each initial point is separated from its immediate neighbours by being enclosed in a polygon, with each face joining the polygons of all its neighbours.

Taking the two procedures attract and repel, we can make a small modification to the attract one, so that instead of turtles being attracted to the constant location 0 0, they are instead interested in another of the turtles acting as a 'target'. Therefore we can make two kinds of turtle – normal ones and targets. Both the normal turtles and the target turtles obey the repel rule, but the attract rule only applies to normal turtles, who try to stay at a particular radius from the target turtles:

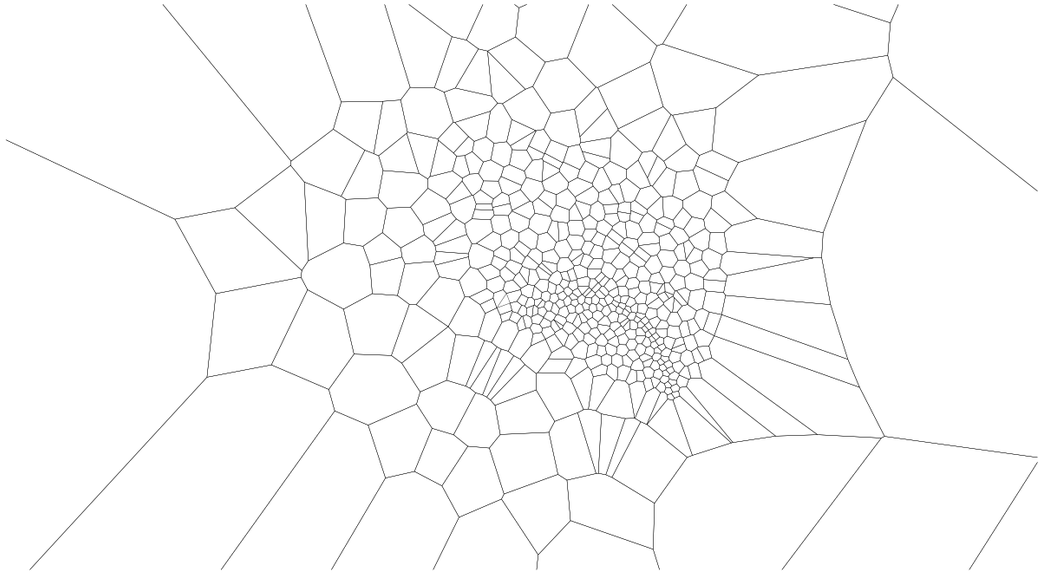
```
to attract
locals [targets]
ask turtles
[
  set targets turtles with [target = true]
  set closest-turtle min-one-of other
      targets [distance myself]
  set heading towards closest-turtle
  ifelse ((distance closest-turtle) <
      radius) [bk 1] [fd 1]
]
end
```

Emergent spatial tessellation of minimal path polygons

In the series above, a very large number of turtles slowly retreat from the stationary targets (larger dots) to form the boundaries of the *Voronoi tessellation*. This is an example of an emergent self-organised structure, where the algorithm goes with the flow of the problem to be solved, namely draw the equidistant boundaries given the initial distribution of points. The answer emerges naturally from the very simple process described above.

The difference between the code for drawing a circle and the code for drawing a Voronoi diagram using the traditional 'computational geometry' approach is huge: the two trig functions described earlier have to be expanded to many pages of code dealing with complex maths and elaborate sorting and scheduling procedures in order to define the polygons, whereas the step from circle to Voronoi using the attract and repel procedures is simply to have two kinds of turtles and a lot more of them!

All this is intended to illustrate the fundamental point about how representational methods can change when we use the Turing machine to generate form. As we shall see in the next section, the complexity of the emergent forms can be much higher than defining them in purely geometric ways. With these two texts we can represent a huge range of objects, and interestingly the representation hardly has to change at all to accommodate the third dimension.



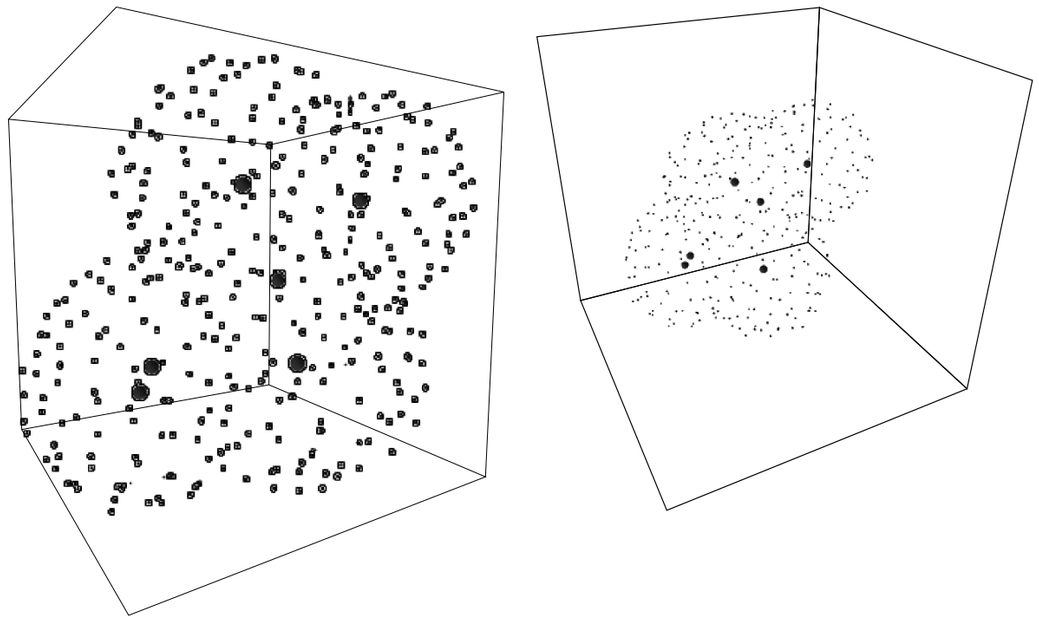
Voronoi by computational ceometry – this was generated as part of an experiment in recursive Voronoi diagrams where each generation provides the seed points for the next diagram

The code on this page can be contrasted with the short snippet on page 15. Both are doing essentially the same thing – generating the minimal path tessellation known as a Voronoi diagram. However, the one on page 15 is written in NetLogo as a parallel process of dynamic systems of turtles, the other is written in BASIC as an exercise in computational geometry (code by the author). Not only is the BASIC enormously longer, but it is also much more restrictive in that it does not allow for easy manipulation of the underlying generating points or alterations of the dynamics of the particles. The only advantage this approach has over the emergent version is that the defined polygons are explicitly defined by ordered line segments, whereas the images taken from the agent-based examples would need a little post-processing to define them.

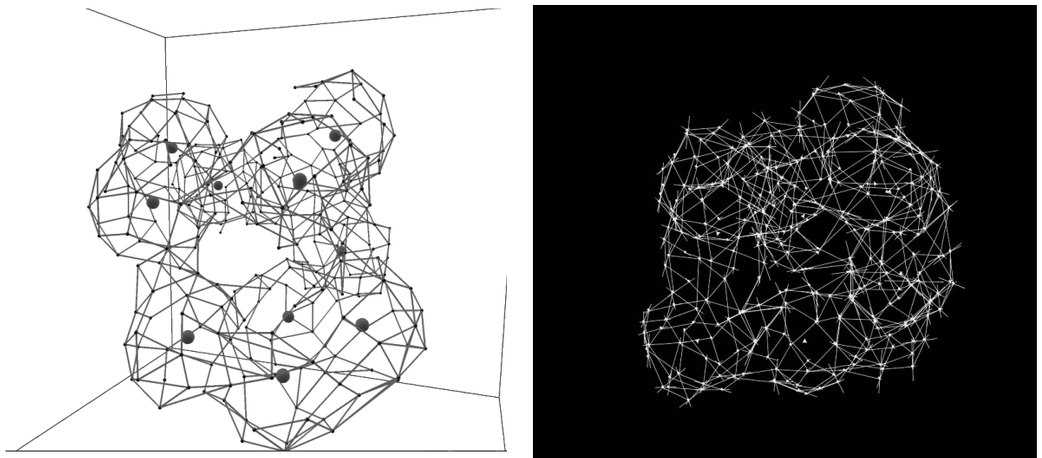
BASIC is a very old programming language used in many Windows applications to automate operations. See Chapter 3 for a discussion of the badness of BASIC.

```
Attribute VB_Name =
  "Voronoibits"

'----------------------
  changing datastructure
  to hold indeces into
  originalpoints
'------------------ rather
  than points 11.6.03-----
  -----------------
' defining the cells of the
  voronoi diagram
' working 26 june 03

Const pi = 3.1415926535
Const yspace = 0
Const xspace = 1

Type pointedge
pos As point 'position of
  intersection
Bedge(2) As Integer
  'indeces into boundary
  array where intersection
  occurs
End Type

Type intersectStuff
outnode As point
outnodeid As Integer
  'index into vertex array
  for voronoi cell
beforeinter As pointedge
afterinter As pointedge
End Type

Const VERYSLOW = 0.7
Type mypoint
x As Double
y As Double
z As Double
spacetype As Integer
kuller As Integer
End Type

Type pair 'to tie the
  triangle nos to the
  sorted angles
value As Double
index As Integer
End Type

Type delaunay
p1 As Integer
p2 As Integer
p3 As Integer
circcentre As mypoint
  'the coordinates of the
  centre of the circle by
  3 pts constructed by
  this point
circrad As Double 'the
  radius of this circle
End Type

Type cell
item() As Integer
tot As Integer
area As Double
id As Long
spacetype As Integer
jump As Boolean
kuller As Integer
End Type

Public pts As Integer
Public numtriangles As
  Integer
Public originalpoints() As
  mypoint
Public triangles() As
  delaunay
Public cells() As cell
Public neighbour() As cell

Public cyclesmax As Long
Public cycles As Long


Sub voronoi(d As Integer)
ReDim cells(1 To pts) As
  cell
ReDim neighbour(1 To pts)
  As cell
Dim i As Integer, j As
  Integer, k As Integer

For i = 1 To pts
cells(i).spacetype =
  originalpoints(i).
  spacetype ' having been
  set in teatime
cells(i).kuller =
  originalpoints(i).kuller
Next i


cycles = 0
numtriangles = 0
'cyclesmax = pts ^ 3

For i = 1 To pts
For j = i + 1 To pts
For k = j + 1 To pts
' the triangles array is
  populated in the sub
  drawcircle - sorry !!
drawcircle_ifnone_inside
  i, j, k, pts
cycles = cycles + 1
'counterform.count_Click
Next k
Next j
Next i

collectcells (0) 'define
  data for all voronoi
  cells
neighcells (0) 'define
End Sub
Sub collectcells(d As
  Integer) ' populates
  array cells with lists
  of all the vertex
  incident triangles of a
  point
Dim v As Integer, N As
  Integer, t As Integer

For v = 1 To pts ' go
  through all the original
  points
N = 0
ReDim cells(v).item(1 To
  1)
' drawpoint
  originalpoints(V),
  acGreen, 2
' ThisDrawing.Regen
  acAllViewports

For t = 1 To numtriangles
  'go through all
  triangles
If triangles(t).p1 = v Or
  triangles(t).p2 = v Or
  triangles(t).p3 = v Then
N = N + 1 '' T is
  index into a tri
  sharing a vertex with
  originalcells(V)
ReDim Preserve cells(v).
  item(1 To N)
cells(v).item(N) = t
cells(v).tot = N
End If
Next t
sortbyangle v, cells(v)
Next v
End Sub
Function centre_
  gravity(this As
  delaunay) As mypoint
Dim tx As Double, ty As
  Double, tz As Double
tx = (originalpoints(this.
  p1).x +
  originalpoints(this.
  p2).x +
  originalpoints(this.
  p3).x) / 3
ty = (originalpoints(this.
  p1).y +
  originalpoints(this.
  p2).y +
  originalpoints(this.
  p3).y) / 3
tz = 0
centre_gravity.x = tx
centre_gravity.y = ty
centre_gravity.z = tz

End Function
```

```
Sub sortbyangle(index As
  Integer, this As cell)
Dim angles() As pair, i As
  Integer, O As mypoint,
  CG As mypoint
ReDim angles(1 To this.
  tot) As pair
O = originalpoints(index)
For i = 1 To this.tot
CG = centre_
  gravity(triangles(this.
  item(i)))
angles(i).value =
  getangle(O, CG)
angles(i).index = this.
  item(i)
Next i
bubblesort angles, this.
  tot
For i = 1 To this.tot
this.item(i) = angles(i).
  index
Next i

End Sub
Sub bubblesort(s() As
  pair, N As Integer)
Dim index As Integer,
  c As Integer, swap As
  Integer, temp As pair

Do
swap = False
For c = 1 To N - 1

If s(c).value > s(c +
  1).value Then
temp = s(c)
s(c) = s(c + 1)
s(c + 1) = temp
swap = True
End If

Next c
Loop Until (swap = False)

End Sub
Function getangle(st As
  mypoint, fin As mypoint)
  As Double

Dim q As Integer, head As
  Double, add As Double
Dim xd As Double, yd As
  Double, r As Double
' calculate quadrant
If fin.x > st.x Then
  If fin.y > st.y Then
q = 1
Else
q = 2
End If
Else
If fin.y < st.y Then
q = 3
Else
q = 4
End If
End If

Select Case q

Case 1
xd = fin.x - st.x
yd = fin.y - st.y
If xd = 0 Then
r = pi / 2
Else
r = yd / xd
End If
add = 0
Case 2
yd = st.y - fin.y
xd = fin.x - st.x
add = 270
If yd = 0 Then
r = pi / 2
Else
r = xd / yd
End If
Case 3
xd = st.x - fin.x
yd = st.y - fin.y
If xd = 0 Then
r = pi / 2
Else
r = yd / xd
End If
add = 180
Case 4
xd = st.x - fin.x
yd = fin.y - st.y
If yd = 0 Then
r = pi / 2
Else
r = xd / yd
End If
add = 90
End Select

If xd = 0 Then
getangle = 90 + add
Else
getangle = ((Atn(r) / pi)
  * 180) + add
End If


End Function
Sub neighcells(d As
  Integer)

Dim v As Integer, N As
  Integer, nbs As Integer,
  cp As Integer

For v = 1 To pts
nbs = 0
  'go through the item
  list for this cell
  (based on vertex V)
For cp = 1 To cells(v).tot
  - 1 'the indeces into
  array cells
N = matchupcells(cells(v).
  item(cp), cells(v).
  item(cp + 1), v) 'two
  points on the voronoi
  region
If N > 0 Then
nbs = nbs + 1
ReDim Preserve
  neighbour(v).item(1 To
  nbs)

  neighbour(v).item(nbs)
  = N
neighbour(v).tot = nbs
End If
Next cp
Next v
End Sub

Function matchupcells(p1
  As Integer, p2 As
  Integer, current As
  Integer) As Integer

' find a cell (in array
  cells)which shares an
  edge p1 - p2 with this
  cell (current)
Dim m As Integer, v As
  Integer, cp As Integer
matchupcells = 0

For v = 1 To pts
If v <> current Then 'dont
  look at you own list
m = 0

'a voronoi region can only
  share two verteces ( one
  edge) with any other
'but since the edges
  are organised anti
  clockwise, the
  neighbouring cell
'will be going the other
  way. so here we just
  look for two matches
  hope thats ok?
For cp = 1 To cells(v).tot
  'run through vertex list
  for this cell
If cells(v).item(cp) = p1
  Then m = m + 1
If cells(v).item(cp) = p2
  Then m = m + 1
Next cp
If m = 2 Then
matchupcells = v
Exit For 'dont go on
  looking once found a
  match
End If
End If
Next v
End Function

Sub drawcircle_ifnone_
  inside(i As Integer,
  j As Integer, k As
  Integer, pts As Integer)
Dim testcircle As delaunay

testcircle.p1 = i
testcircle.p2 = j
testcircle.p3 = k
circbythreepts testcircle
If Not inside(testcircle,
  pts) Then
'drawpoint testcircle.
  circcentre, acYellow,
  testcircle.circrad
numtriangles =
  numtriangles + 1
ReDim Preserve triangles(1
  To numtriangles)

  triangles(numtriangles)
  = testcircle
End If

End Sub

Function inside(this
  As delaunay, pts As
  Integer) As Integer
' are there any points
  closer to the centre of
  this circle than the
  radius

inside = False
Dim i As Integer, dd As
  Double, cr As Double
For i = 1 To pts
'ignore points that are on
  this circle
If i <> this.p1 And i <>
  this.p2 And i <> this.
  p3 Then
dd = distance(this.
  circcentre,
  originalpoints(i))
cr = this.circrad
If (dd < cr) Then
inside = True
Exit For
End If
End If
Next i
End Function
Sub circbythreepts(this As
  delaunay)

Dim a As Double, b As
  Double, c As Double, k
  As Double, h As Double,
  r As Double, d As
  Double, e As Double, f
  As Double
Dim pos As mypoint
Dim k1 As Double, k2 As
  Double, h1 As Double, h2
  As Double

a = originalpoints(this.
  p1).x: b =
  originalpoints(this.
  p1).y
c = originalpoints(this.
  p2).x: d =
  originalpoints(this.
  p2).y
e = originalpoints(this.
  p3).x: f =
  originalpoints(this.
  p3).y

'three points (a,b),
  (c,d), (e,f)
'k = ((a²+b²)(e-c) +
  (c²+d²)(a-e) + (e²+f²)
  (c-a)) / (2(b(e-c)+d(a-
  e)+f(c-a)))
k1 = (((a ^ 2) + (b ^ 2))
  * (e - c)) + (((c ^ 2)
  + (d ^ 2)) * (a - e)) +
  (((e ^ 2) + (f ^ 2)) *
  (c - a))
k2 = (2 * ((b * (e - c))
  + (d * (a - e)) + (f *
  (c - a))))
k = k1 / k2

'h = ((a²+b²)(f-d) +
  (c²+d²)(b-f) + (e²+f²)
  (d-b)) / (2(a(f-d)+c(b-
  f)+e(d-b)))
h1 = (((a ^ 2) + (b ^ 2))
  * (f - d)) + (((c ^ 2)
  + (d ^ 2)) * (b - f)) +
  (((e ^ 2) + (f ^ 2)) *
  (d - b))
h2 = (2 * (((a * (f - d))
  + (c * (b - f)) + (e *
  (d - b)))))
h = h1 / h2

'the circle center is
  (h,k) with radius; r² =
  (a-h)² + (b-k)²
r = Sqr((a - h) ^ 2 + (b -
  k) ^ 2)

pos.x = h: pos.y = k:
  pos.z = 0
''drawpoint pos, acYellow,
  r
this.circcentre = pos
this.circrad = r

End Sub

Sub convert(b As mypoint,
  f As mypoint, start()
  As Double, finish() As
  Double)

start(0) = b.x
start(1) = b.y
start(2) = b.z
finish(0) = f.x
finish(1) = f.y
finish(2) = f.z
End Sub

Function findcenter(pts As
  Integer) As mypoint
Dim xt As Double, yt As
  Double

xt = 0
yt = 0

For i = 1 To pts
xt = xt +
  originalpoints(i).x
yt = yt +
  originalpoints(i).y
Next i

findcenter.x = xt / pts
findcenter.y = yt / pts
findcenter.z = 0

End Function

Sub Draw_Line(b As
  mypoint, f As mypoint, c
  As Integer)
Dim lineobj As AcadLine
Dim mLineObj As AcadMLine
Dim start(0 To 2) As
  Double, finish(0 To 2) As
  Double

convert b, f, start, finish

Set lineobj =
  ThisDrawing.ModelSpace.
  AddLine(start, finish)

lineobj.color = c
lineobj.Layer = "delaunay"
'lineobj.Update

End Sub
Sub drawpoly(this As cell)
Dim tri As delaunay
Dim plineObj As
  AcadLWPolyline
'changed to lw polyline so
  only duets of coords not
  trios
Dim thepoly(0) As
  AcadEntity 'thing to use
  in addregion
Dim boundary As Variant
  'assign with addregion
Dim boundy() As AcadRegion
  'thing you redim
Dim acell As AcadRegion
Dim numtri As Integer,
  thepoints() As Double,
  TPC As Integer
numtri = this.tot * 2 - 1
ReDim thepoints(numtri +
  2) As Double
TPC = 0
' loop through all the
  items getting the
  coordinates of the
  circlcentres that are
' inside the elements of
  the thetriangles array

For i = 1 To this.tot
thepoints(TPC) =
  triangles(this.item(i)).
  circcentre.x
TPC = TPC + 1
thepoints(TPC) =
  triangles(this.item(i)).
  circcentre.y
TPC = TPC + 1
' thepoints(TPC) =
  triangles(this.item(i)).
  circcentre.z
' TPC = TPC + 1
Next i
thepoints(TPC) =
  thepoints(0)
TPC = TPC + 1:
  thepoints(TPC) =
  thepoints(1)
'TPC = TPC + 1:
  thepoints(TPC) =
  thepoints(2)

If TPC > 3 Then
On Error Resume Next
  'got crash on huge poly
Set plineObj =
  ThisDrawing.ModelSpace.A
  ddLightWeightPolyline(th
  epoints)
If plineObj.area > 0 Then

Set acell =
  makeregion(plineObj)

On Error Resume Next
acell.Boolean
  acIntersection, bound
this.area = acell.area
this.id = acell.ObjectID
  'changed to acell
If this.spacetype = 1 Then
acell.color = this.kuller
Else
acell.color = acWhite
End If

' acell.Update
'  ThisDrawing.Regen  acAc-
  tiveViewport
makeboundaryregion 0
End If

End If

End Sub

Sub drawcircle(x As
  Variant, y As Variant,
  kuller As Integer, size
  As Integer)
Dim p(2) As Double, circ
  As AcadCircle
p(0) = x: p(1) = y: p(2)
  = 0
Set circ = ThisDrawing.
  ModelSpace.AddCircle(p,
  size)
circ.color = kuller
' circ.Update

End Sub
Function random(bn As
  Double, tn As Double) As
  Double

random = ((tn - bn + 1) *
  Rnd + bn)

End Function

Function distance(startp
  As mypoint, endp As
  mypoint) As Double
Dim xd As Double, yd As
  Double
xd = startp.x - endp.x
yd = startp.y - endp.y
distance = Sqr(xd * xd +
  yd * yd)
End Function

Sub drawpoint(pos As
  mypoint, c As Integer, r
  As Double)
' This example creates a
  point in model space.
Dim circleObj As
  AcadCircle
Dim location(0 To 2) As
  Double
location(0) = pos.x
location(1) = pos.y
location(2) = pos.z
' Create the point
Set circleObj =
  ThisDrawing.ModelSpace.
  AddCircle(location, r)
circleObj.color = c
'ZoomAll
End Sub
```



(*above*) From rings of points to spherical clouds
(*below*) Using a link turtle to join the dots



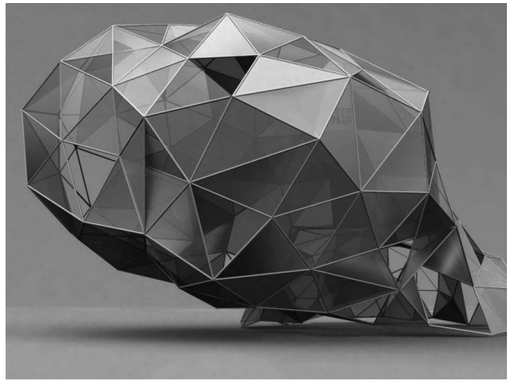
Moving into the third dimension

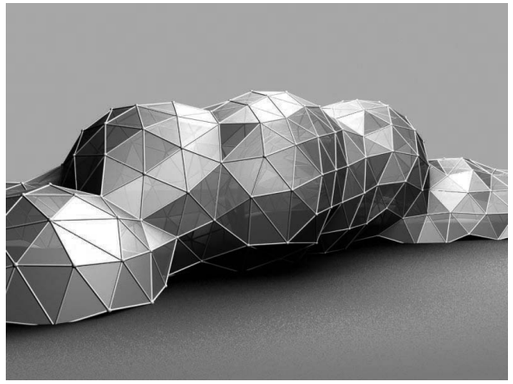
The code below is pretty much the same as before (there are a few differences due to the 3D version of the language being a revision behind the 2D version, but we can ignore those), apart from that the only difference is the use of the word `pitch` as well as `heading`, which allow the turtles to point towards things in 3D space:

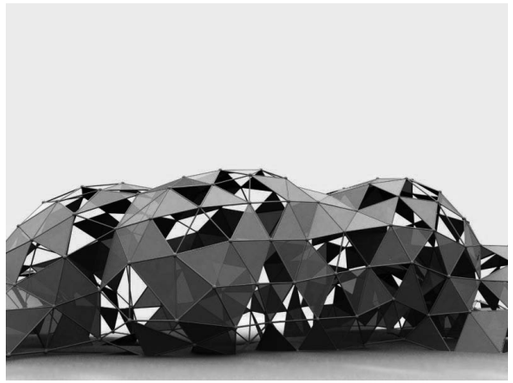
```
to attract
ask nodes
[
     set closest-turtle min-one-of targets
       with other targets [distance
       myself]
     set heading towards-nowrap closest-
       turtle
     set pitch towards-pitch-nowrap
       closest-turtle
     ifelse ((distance closest-turtle) <
       radius) [bk 1] [fd 1]
]
end

to repel-nodes
ask nodes
[
     set closest-turtle min-one-of nodes
       with other nodes [distance myself]
     set heading towards-nowrap closest-
       turtle
     set pitch towards-pitch-nowrap
       closest-turtle
     bk repel-strength
]
end
```

When running these simulations another thing that distinguishes this approach from geometry becomes apparent: rather than in the top-down computational approach, where a lot of works goes on until the 'solution' is presented to you in one fell swoop, here the emergent organisation occurs as a visible process that sometimes has to be teased along with small tweaks of attract and repel values. Sometimes the whole thing descends into a chaotic muddle and cannot be retrieved without stopping and starting again. The algorithm for stitching the turtles together with line-shaped turtles is typical of the bottom-up approach.

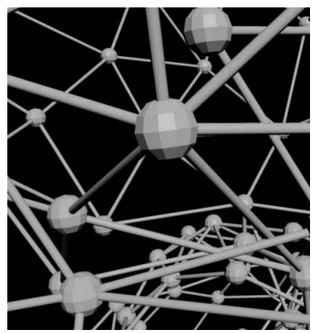
Once the closest turtle has been found, we ask each node to create a link with it. The 'link' turtle is a special feature of NetLogo which behaves intelligently in that if the target node is already connected, then this is not attempted again. In the course of a run, the 'nearest turtle' will change so it is necessary to clear out existing links – this is easily accomplished with '`clear-links`' (a special button – not shown – is needed for this).

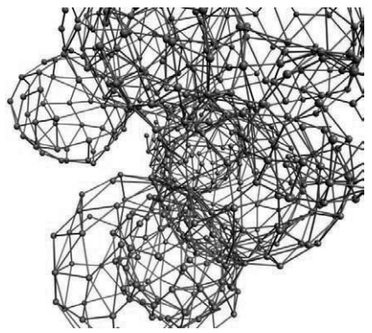






Further processing to develop the emergent distributions into varieties of forms






```
ask nodes
[
     set closest-turtle min-one-of other
       nodes [distance  myself]
     set heading towards-nowrap closest-
       turtle
     set pitch towards-pitch-nowrap
       closest-turtle
     bk repel-strength
     create-link-with closest-turtle
]
end
```

One might ask why this simple algorithm does not lead to links which cross the middle of the emerging spheroid, but remember that the attract and repel procedures have a habit of making sure that everyone's nearest neighbour is to be found on the 'shell'. Where several spheroids meet (as in the images on the facing page), a certain amount of negotiation takes place, with things jiggling about until most people are happy. The important point here is that no more code has to be written, this is an emergent outcome of the process provided for free by the dynamics of the system.

After everything has settled down (the 'emergent consensus' proposed at the start of this chapter), the self-organised turtle configurations can be exported to other packages for further processing. In the images shown on the facing page, the turtle coordinates are read into AutoCAD using a small Visual Basic script, and spheres and cylinders are drawn between the points of the nodes and links. Further processing to tile up the mesh and rendering can be achieved with your favourite CAD package.