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# HYDRAULIC DESIGN CHARTS FOR ELLIPTICAL PIPES

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**Abstract:** This paper concerns charts for the hydraulic design of elliptical pipes under part-full gravity flow conditions. Examples of such pipes are currently commercially available, and may be laid with the major axis of the cross section either horizontal or vertical. Similar charts have been published for circular pipes and for pipes of ovoid cross section, which have particular uses in combined sewerage. Such charts are useful for preliminary sizing and outline design purposes. The applications of this type of chart to elliptical pipes lead to two distinctive points that will be discussed in the paper. Firstly, the two possible orientations of the pipe resulted in different sets of lines on the chart, and these may be used to illustrate the relative performance of the two orientations, which have different practical benefits. The second point, of somewhat more academic interest, relates to the geometry of the ellipse. Although the cross sectional area of the ellipse has a well known and exact mathematical formula, the perimeter of the ellipse does not. Different approximate expressions for the total perimeter are compared with the elemental approach that was used to establish the pipe cross section geometry for the computerised production of the design charts.

## 1. Introduction:

Although the circle is the most common pipe cross sectional shape, a number of other cross sections are also available particularly for gravity flow applications. These range from rectangular box culverts, to the ovoid shape common in old brick built combined sewers. Elliptical pipes are also produced, in the UK for example by Stanton Bonna (2002), and these can provide a number of useful properties, depending on whether they are laid with the major axis horizontal or vertical. When laid with the major axis horizontal, the pipe provides high flow capacity for a relatively shallow depth of construction. With the major axis vertical, the shape of the invert provides a better velocity at part-full low flow rates than would be obtained from a circular pipe.

Design charts have their uses, particularly for preliminary sizing and to assist hand calculations, and although tables are available covering elliptical shapes, no charts have been published. The production of design charts for part-full flow in elliptical pipes provided an interesting challenge for a postgraduate student, requiring understanding of the hydraulics, geometry and computational issues. This paper presents the main points involved, and contains examples of the charts produced.

## 2. Hydraulics:

The hydraulic design considers non pressurised uniform flow, with the water surface parallel to the pipe invert, and hence the hydraulic gradient equal to the pipe gradient. The established Colebrook-White friction formula is used to express average velocity V as a function of gradient S, roughness height k and hydraulic radius R:

$$V = -2\sqrt{8gRS}\,\log\left(\left(\frac{k}{14.8R}\right)+\left(\frac{2.51\nu}{4R\sqrt{8gRS}}\right)\right) \tag{1}$$

where acceleration due to gravity $g = 9.81$ m/s$^2$, and kinematic viscosity $\nu = 1.14 \times 10^{-6}$ m$^2$/s for water at 15$^o$C. This assumes that replacement of diameter by 4R is appropriate in this context. The hydraulic radius R depends on the depth of flow in the part-full elliptical pipe, and is calculated from the area A and the wetted perimeter P

$$R = \frac{A}{P} \qquad (2)$$

Concerning values for surface roughness k, BS EN 752 (2008) states that values currently in use range from 0.03 mm to 3.0 mm for pipe sewers, although the lower values seem unlikely for sewers in use. The national annex providing guidance for the use of this standard in the UK recommends a pipeline roughness value of 0.6 mm, rising to 1.5 mm for foul and combined sewers with peak flow velocity less than 1.0 m/s. Note that although often quoted in mm, the value of k in equation (1) should be expressed in metres (m) consistent with other units.

## 3. Geometry of the Ellipse:

The cross section area A of the full ellipse is well known, in terms of the semi major axes a and b

$$A = \pi ab \qquad (3)$$

The perimeter P however does not have an exact expression. Rosin and Pitteway (2001) quote approximations to the perimeter, the first $P_1$ being described as a relatively crude approximation

$$P_1 = \pi\sqrt{2(a^2 + b^2)} \qquad (4)$$

The second $P_2$ is described as a more effective approximation, attributed to Ramanujan

$$P_2 = \pi\left(3(a+b) - \sqrt{(3a+b)(a+3b)}\right) \qquad (5)$$

Other more involved expressions are available, but were found not to differ significantly for this application (with aspect ratios in the range $1.5 < a/b < 1.8$) from values given by equation (5).

The above expressions are all for the full perimeter of the ellipse, and there is no easy way of finding the length of part of an ellipse, such as can be used for the length of an arc of a circle.

Since part-full properties were required at various depths, these were built up from 1 mm thick slices, each treated as a trapezium with the area calculated accordingly, and the wetted perimeter taken as the sloping side lengths. The results of this elemental method of slices were checked, as shown for example in Table 1 for the smallest available pipe size, and found to be in good agreement certainly more than sufficient for practical purposes.

Elliptical pipe 1.000 m x 0.650 m internal
a = 0.500 m
b = 0.325 m
A = 0.510476 m$^2$ (method of slices)
A = 0.510509 m$^2$ (Equation 3)
P = 2.6210 m (method of slices)
$P_1$= 2.6495 m
$P_2$= 2.6211 m

Table 1: Ellipse geometry data

It can be seen from Table 1 that the area figure from the method of slices agrees to four significant figures with the precise value from equation (3), with the error being less than 0.01%. The perimeter value by the method of slices agrees to the nearest mm with equation (5). It is noticed that the approximate value $P_1$ overestimates by about 1%, and the method of slices is more accurate than this approximate formula.

## 4. Design Charts:

Design charts for part-full flow in circular pipes produced by Butler and Pinkerton (1987) provided advantages when compared with the well known proportional depth chart for flow in circular sections. This approach was applied to ovoid pipes by Marriott (1996, 2001). This format has the advantage of incorporating both full and part full flows on one chart, although a separate chart is needed for each pipe size and each roughness considered. This was the form of design chart used for elliptical pipes by Uddin (2007), who produced charts using Microsoft Excel for three sizes with five roughness values (0.15, 0.3, 0.6, 1.5 and 3.0 mm), and covering both possible orientations, amounting to a set of 30 different charts. Sizes of elliptical pipe available from Stanton Bonna range from 1.000 m x 0.650 m up to 2.650m x 1.500 m, and Uddin covered those two sizes and one intermediate size of 1.650 m x 1.000 m. Examples of the charts are shown in Figures 1, 2, 3, and 4 for this intermediate size, laid in either horizontal or vertical orientation, with roughness values of 0.6 mm and 1.5 mm. An additional illustration in Figure 5 shows a comparison of part-full performance for the pipe laid in horizontal and in vertical orientation, which illustrates the extent of the velocity advantage at low depths by using the vertical orientation. In this example, at a proportional depth of 0.1 the vertical orientation shows over 25% greater velocity than the horizontal orientation, and the vertical orientation achieves an acceptable self cleansing velocity of 0.7 m/s, whereas the horizontal orientation does not.

## 5. Conclusions:

A set of hydraulic design charts for part-full flow in elliptical pipes was successfully produced, and examples have been given in this paper.

The charts illustrate the velocity advantage at low proportional depths of the vertical orientation of the elliptical pipe, which can amount to over 25%. This may be advantageous in achieving self cleansing conditions, although the horizontal orientation has practical benefits in limiting the depth of construction required.

The areas and wetted perimeters for the ellipse produced by an elemental method of slices agreed well with established formulae, in particular the Ramanujan approximation for perimeter, and showed that a simpler approximate formula for perimeter overestimates the length of the perimeter.

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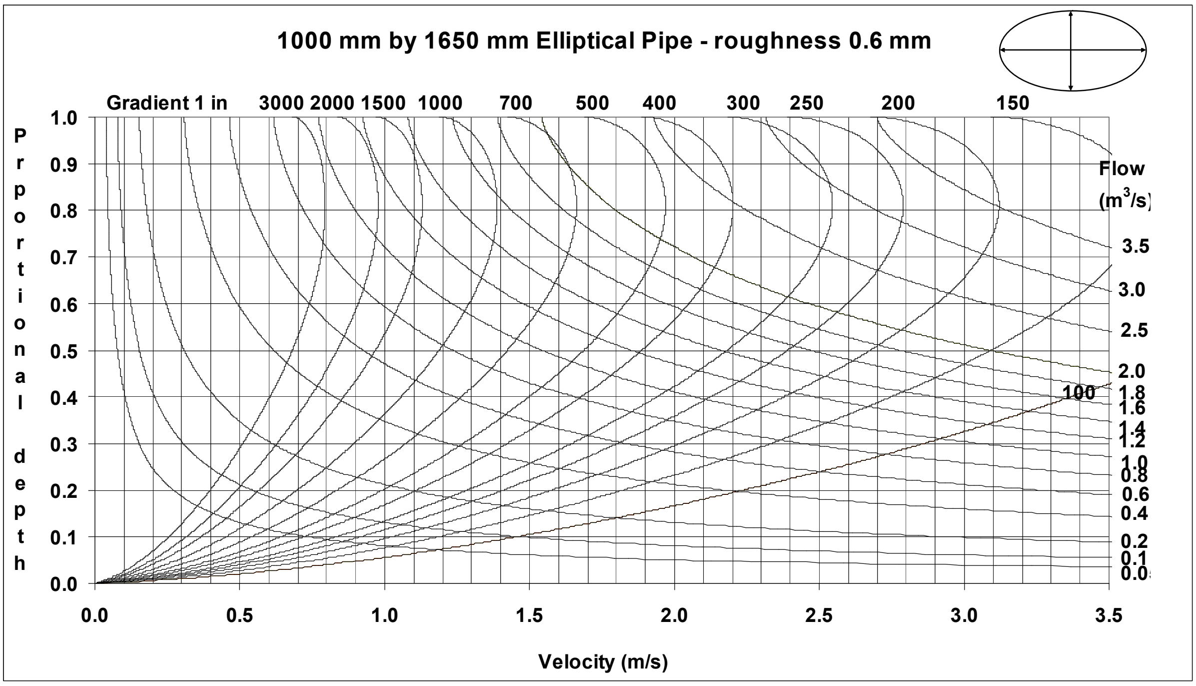

Figure 1: Horizontal orientation, roughness 0.6 mm

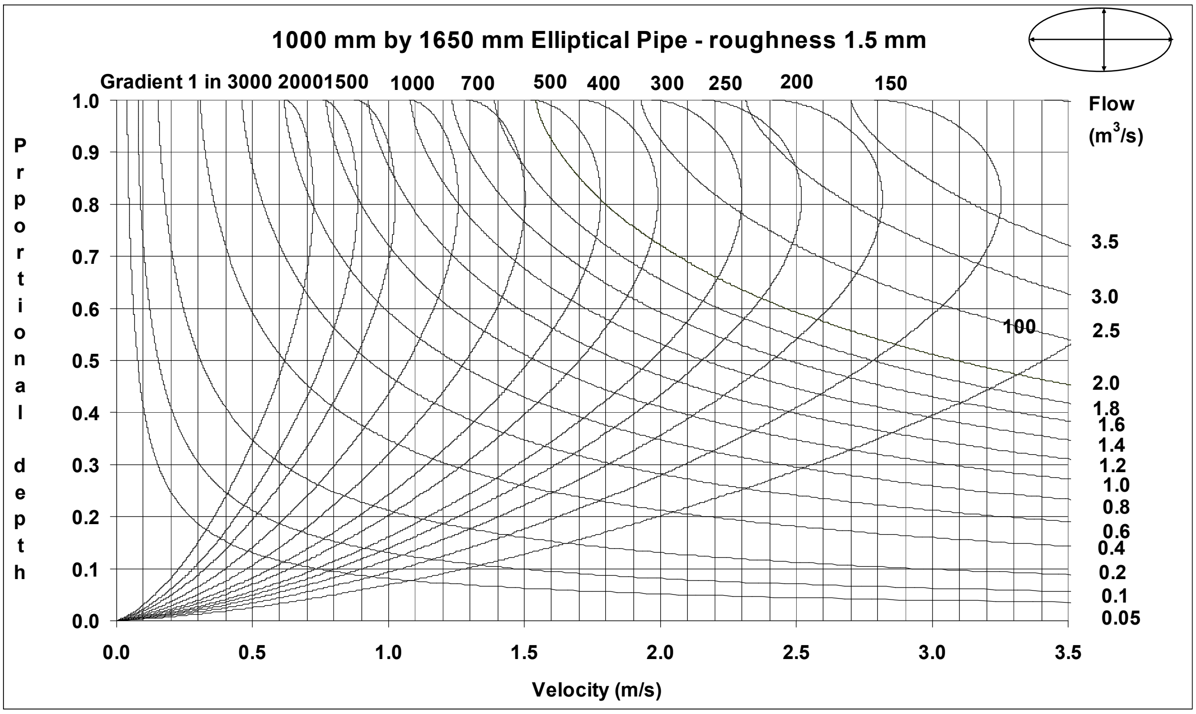

Figure 2: Horizontal orientation, roughness 1.5 mm

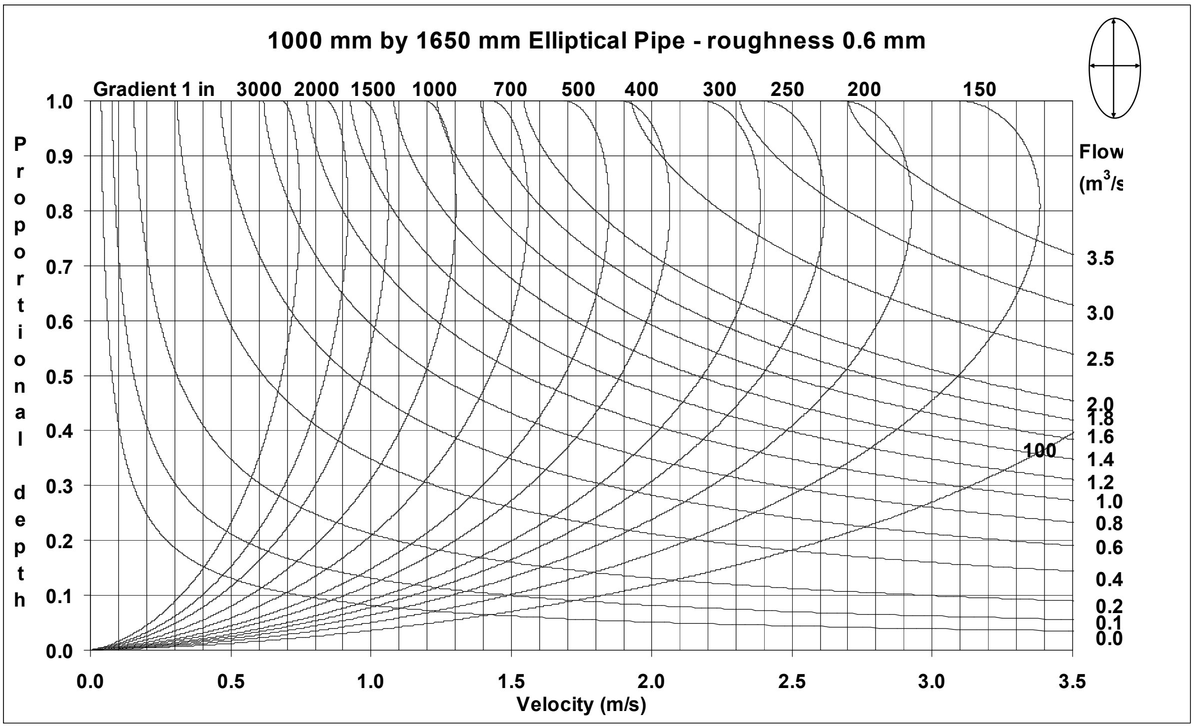

Figure 3: Vertical orientation, roughness 0.6 mm

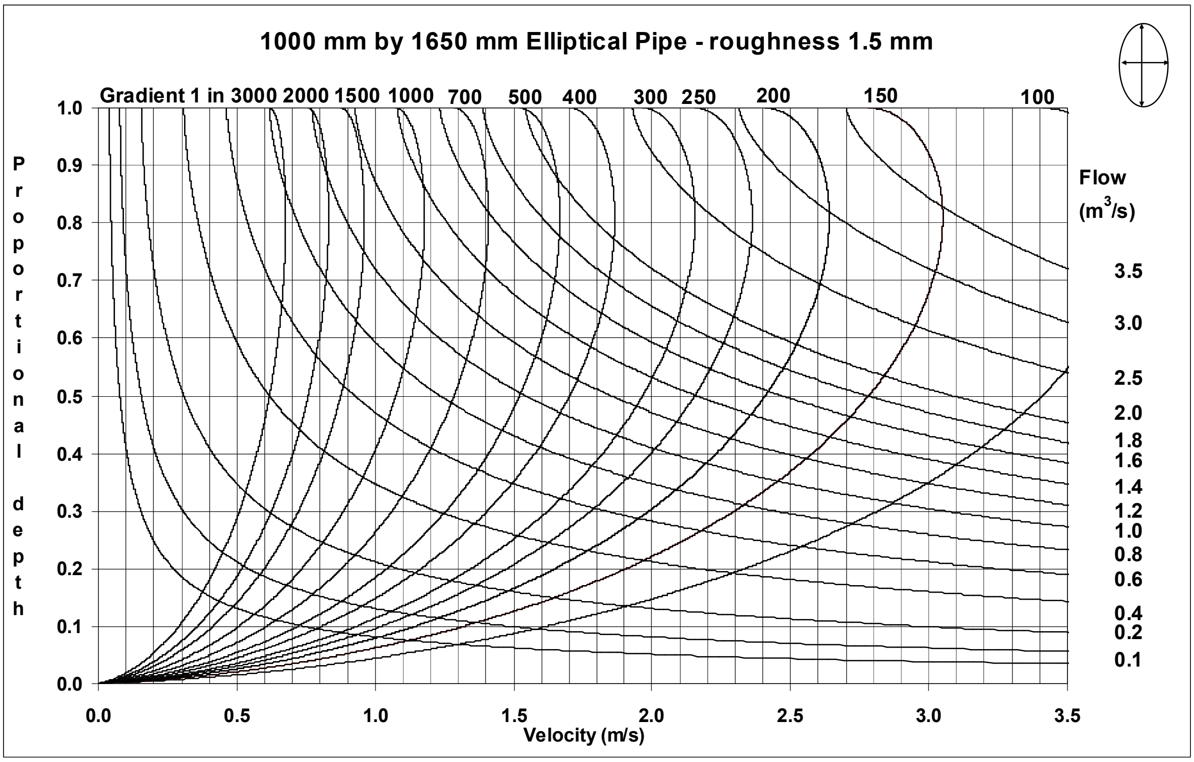

Figure 4: Vertical orientation, roughness 1.5 mm

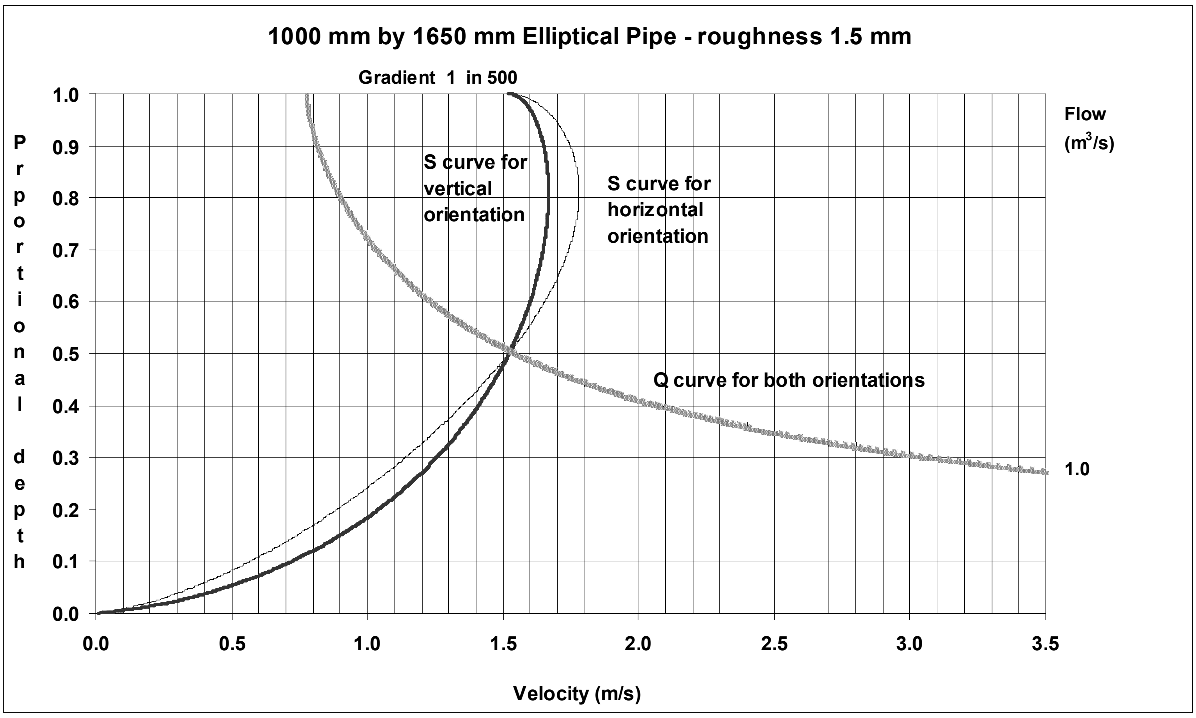

Figure 5: Comparison of horizontal and vertical orientations, roughness 1.5 mm