# PHOTOELASTIC METHOD REVISITED FOR RESIDUAL STRESS MEASUREMENT IN HIGH DENSITY POLYETHYLENE PIPES

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**Abstract:** Residual stresses, also called "frozen-in" stresses in high-density polyethylene (HDPE) pipes, result from uneven cooling that occurs between the inner and outer surfaces during the extrusion process in manufacture. Most known methods of measurements involve intrusive testing to determine the magnitude and profile of the residual stresses in the pipe wall thickness. The Photoelastic Coating Technique (Photostress Method$^{TM}$) described in this paper involves capturing the locked in stresses with minimal interference and local disturbance as the residual stress is released. The existing methods used intrusive testing to determine the residual stress profile whereas the test recommended to WIS 4-32-03 gives a single residual stress value at the bore. The Photostress Method permits stress measurements to be made directly at different points in the pipe wall and a well-defined residual stress profile was consequently obtained. An idealised equation for the residual stress profile was found it was postulated that the residual stress profile in the pipe wall can be determined once the residual stress at the pipe bore was obtained using either the WIS method or the Photostress Method.

## 1. Introduction:

Residual stresses in a polymer develop in plastic pipes during their fabrication by either extrusion or injection moulding. Non-homogeneity of flow and temperature gradients are generally accepted as the main causes of residual stresses in extruded pipes. In extruded high-density polyethylene pipes, heat is dissipated almost exclusively from the exterior surface. Consequently the exterior surface of the pipe is solidified in the early stages of cooling while the inner surface remains in the molten state. When the inside subsequently cools, its surface layer endeavors to contract as a consequence of the thermal change in length, but is prevented from doing so by the external surface layer which has cooled sooner and already assumed its form (Janson, 1999). This differential cooling causes a self-equilibrating residual stress profile to be set-up in the pipe wall with the cessation of the cooling. These residual stresses which can be up to 3MPa are significant (37%) compared to the accepted working stresses (circa 8MPa) in HDPE (PE100). The current techniques of residual stress measurements are inherently locally intrusive and can lead to non-representative residual stress profiles. Therefore, an alternative methodology is desired where the parent material at the local point of interest is left undisturbed and unmodified. The photoelastic coating technique is simple and the sample preparations to obtain readings over a pipe wall thickness is relatively less time consuming. Stresses are measured directly by observing the number of fringes developed on a photoelastic coating adhered to the pipe wall section with an appropriate recommended (resin/hardener) two-part adhesive. The fringes formed are in the hoop direction. The radial stresses are clearly zero at the internal and external surfaces they cannot be significantly different elsewhere (Guan 1993). The stress field will therefore be in the plane of the pipe cross-section and will vary from biaxial compression on the external surface to biaxial tension on the internal surface. The photoelastic coating technique facilitates the stress profile in the pipe wall to be determined on a single pipe specimen.

## 2. Proposed Photoelastic Method:

The proposed method adopts the Photostress Method™ also called the Photoelasic Coating Technique (Zandman et al, 1977) which has the unique capability of measuring:

- the directions of the principal strains and stresses
- the magnitude and sign of the tangential principal stress (the only non-zero principal stress) along free (unloaded) boundaries and in regions where the state of stress is uniaxial and
- the magnitude of the difference in principal strains and stresses in a biaxial stress state.

In this method, strain measurements are made by reflecting polarised light from the stressed surface which has a photoelastic coating adhered to the wall thickness. The surface strains from the stressed part are intimately transferred to the coating since the coating is uniformly bonded on the surface.

The principal stress difference in the structure in term of principal strains is given by:

$$\sigma_x - \sigma_y = (\varepsilon_x - \varepsilon_y) E/(1+\nu) = N f E/(1+\nu) \quad \textbf{Equation 1}$$

where $E$ is the time-dependent Young's Modulus,
$\nu$ is the Poisson's Ratio,
$\varepsilon_x$ , $\varepsilon_y$ are principal strains in x and y directions,
$N$ is the fringe order read from the polariscope and
$f$ is the fringe value obtained from the plastic coating applied.
A boundary condition (at edge) defines one of the principal stresses to be zero. Therefore,

$$\sigma = N f E/(1+\nu) \quad \textbf{Equation 2}$$

where $\sigma$ is the principal stress in direction tangential to the edge.

### 2.1. Photostress Measurement Technique and Experiments:

The authors used a reflective polariscope (030 Series Modular Reflection Polariscope supplied by Vishay Measurement Group, UK). This consisted of a polarised light source, a polariser and analyser plates (Figure 1). A compensator was used with the polariscope to measure a photoelastic signal at a point. The compensator adds to the light path an identical calibrated signal equal in size but opposite in sign. In doing so, the photoelastic signal at a point was completely cancelled to read zero fringe and the colour black was restored at the point. The compensator reading was then taken at this point to calculate the fringe order N.

A photoelastic coating (PS-4A Type, obtained from Vishay Measurement Group, UK) was glued over the thickness of the pipe at the crown of the HDPE pipe (180mm diameter). The typical sample size tested was 180mm long (equivalent to

Diameter of pipe) as recommended in the WIS Standard Test Method.

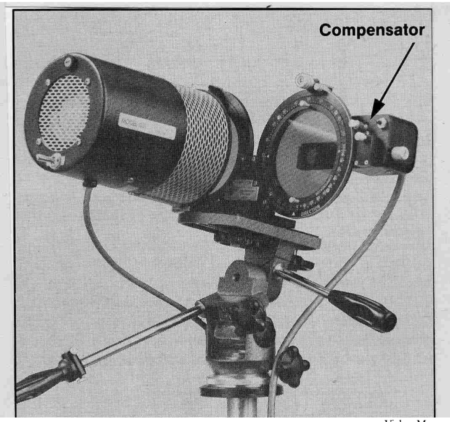


Figure 1: 030 Series Modular Reflection Polariscope

**2.2. Measurement of Principal Strains and Stresses:**

A 180mm long PE100 pipe sample was cut longitudinally into two equal halves at 90 degrees from the crown to release the residual stresses. After the release, readings were taken after one hour and subsequently on a daily basis when more fringes were developed. The fringes were better defined when a high proportion of the residual stresses was released after approximately 7 days. Readings were taken at 2mm spacing across the pipe wall thickness of 17.6mm. The coating was graduated using a reticule at 2mm intervals from the bore (internal pipe surface) of the pipe for ease and accuracy of measurements.

A number of tests were carried-out to check that the method is reliable and that the results could be repeated. Figure 2 shows a typical example of the fringes observed through the polariscope at one hour time period after the residual stresses were released.

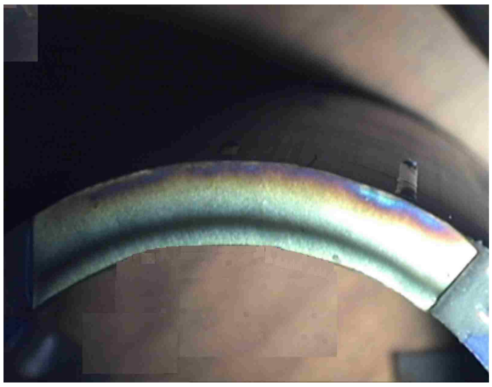

Figure 2:Fringes formed 1 hour after release of residual stress

The residual stresses were calculated from the number of fringes observed at different depths of the pipe wall thickness using Equation 2. It is to be noted that the calculations of the stresses are based on the fringe constant (f) of the photoelastic coating, the Poisson Ratio ($\nu$) and Young's Modulus (E) of the PE100 pipe. Whereas the first two parameters are constant, the Young's Modulus value is dependent on the creep stress level of the material and is time dependent. Hence, the Young's Modulus values in the above experiments were based from experimental creep data for PE100 samples as described in WIS 4-32-05, April 1986.

The experimental results for 180mm long, 180mm diameter SDR11 PE100 pipes are shown in Figure 3. The residual stresses at the bore were in the range of 2.8 to 3.07MPa (tensile) and -2.63 to -2.93MP (compressive) at the exterior surface of the pipe. The flexural (Bending) Creep Modulus used for PE100 at 1 Hour is 519.5 MPa.

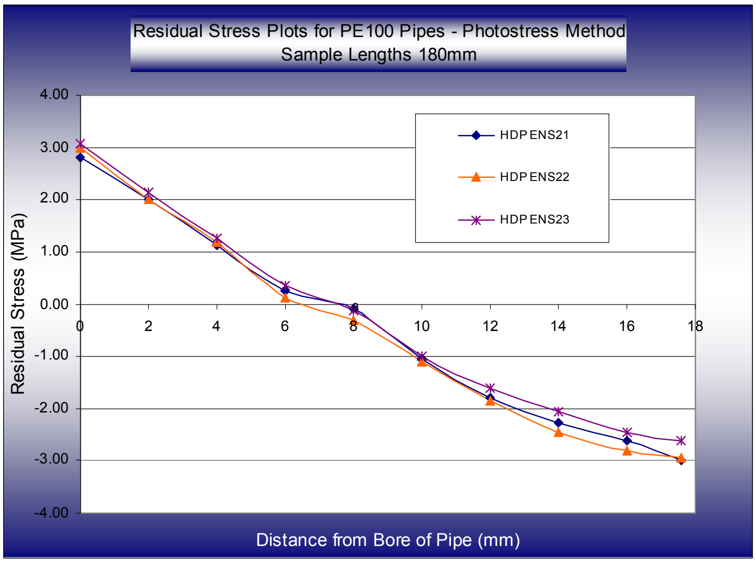


Figure 3: Typical residual stress profile observed across the pipe wall thickness using Photostress Method for 180mm Diameter, SDR11 PE100 Pipes.

## 2.3. Simplified Test Method – 30mm Sample size:

In order to simplify the testing method the authors devised a testing method to ease sample preparation and to ensure that the longitudinal residual stresses in the samples are released significantly and the latter effects on the hoop residual stresses minimised. The authors proposed to reduce the pipe sample length from the WIS recommended pipe length equivalent to the diameter of the pipe (of 180mm) to 30mm. The main benefit derived from this sample length is that the samples can be prepared in lesser time and is easier to handle (being less bulky) and to store.

The authors devised an equivalent residual stress profile based on the 30mm length of the samples and worked out the relationship between residual stress readings from the 180mm and 30mm lengths of samples.

Figure 4 shows the shape of the residual stress profile for the two sample lengths to be similar. However, the shape of the profile is different when comparing with the conventional profiles suggested by other previous authors. Using the two set of results, namely from the 30mm and 180mm samples, the authors worked out an equation which can be used to determine the equivalent residual stresses in 180mm length samples of pipes using the results from the 30mm length of samples. The relationship used is as follows:

$$S_x(180mm) = -0.1019 * S_{30mm}^3 + 0.0563 * S_{30mm}^2 + 1.798 * S_{30mm} - 0.197$$

**Equation 3**

$R^2 = 0.9972$

The correlation coefficient for the Equation 3 was $R^2$= 0.9972 which shows the goodness of fit between the two set of data for the 30mm and the 180mm samples.

In order to correlate the data recorded for the two sample lengths, the authors proceeded with further analyses using dimensionless parameters.

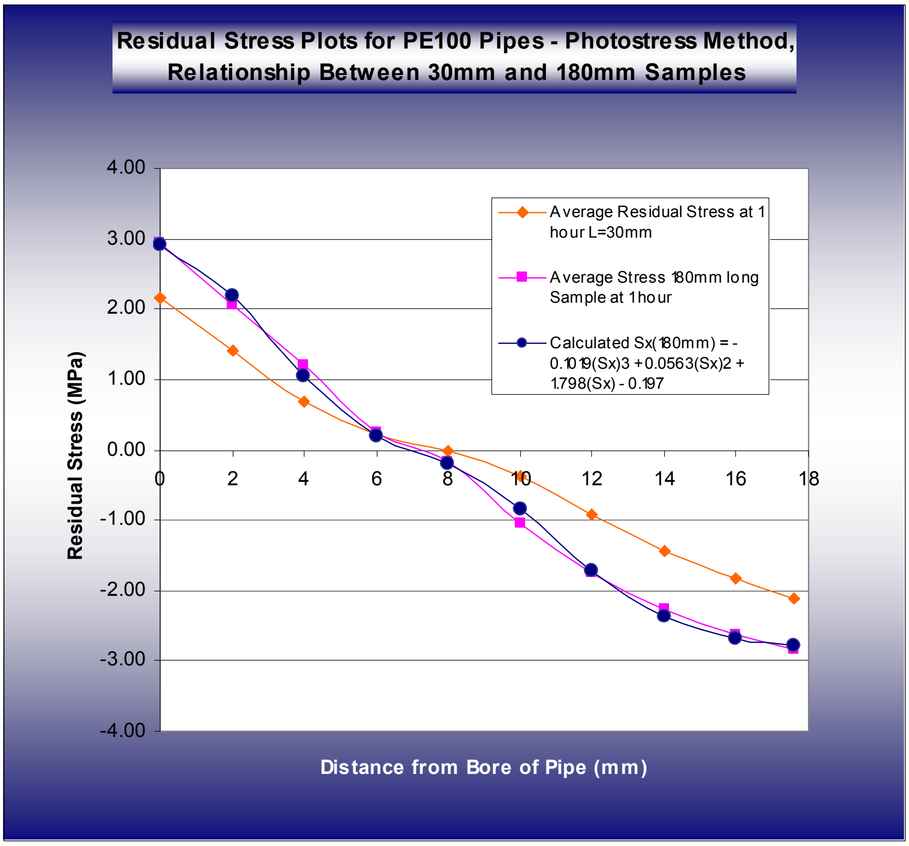


Figure 4: Comparison of Residual Stress Plots Using 30mm and 180 Lengths of Samples.

## 3. Idealised General Equation for Hoop Residual Stress Profile:

The plots of the residual stresses shown so far have been different from the residual stress profiles determined by previous authors. Following the tests described in the previous section the authors plotted the residual stresses obtained from both lengths of samples i.e. the 30mm and 180mm samples as dimensionless parameters as shown in Figure 5 below. The dimensionless plots would help to determine the following:

1. The shape of the residual stress profile across the pipe wall thickness.
2. The magnitude of the residual stresses across the pipe wall provided the residual stress at the bore can be determined.

The dimensionless ratio used in the vertical axis was the residual stresses measured across the pipe wall thickness to the residual stress measured at the bore of the pipe i.e. $S_x/S_{interior}$. The dimensionless ratio used in the horizontal axis was the ratio of distance from the bore at which the residual stress was measured to the thickness of the pipe wall.

It is to be noted that a number of tests were done using the 30mm lengths (11 Nos.) and the 180mm lengths (7 Nos.) of samples. From the plots of each of the two sets of plots idealised trend-lines were obtained using an Excel spreadsheet. These trend-lines were well within the 95% confidence lines and had the R squared values (goodness of fit) of 0.997 for the 180mm length of samples and 0.991 for the 30mm length of samples. Since the parameters

plotted were dimensionless it was not necessary to find proportionality constants for the 180mm and 30mm lengths of pipe samples to compare the two set of results.
An idealised plot of the residual stress profile for the 30mm length of sample was found and an idealised equation as shown in Figure 5 is as follows:

$$\frac{S_x}{S_{internal\ face}} = 0.4489 * (\frac{x}{t})^2 - 2.3499 * (\frac{x}{t}) + 0.9295$$

$R^2$ =0.991 **Equation 4**

where $x$ = distance from the bore of the pipe,
$t$ = thickness of pipe wall,
$S_x$ = calculated residual stress and
$S_{internal\ face}$ = residual stress measured using the Photostress Method.
The above Equation 4 helps in determining the profile of the residual stress across the pipe wall thickness provided the residual stress at the internal face is determined first. The Photostress Method can be used to measure the residual stress at the pipe internal face (pipe bore). Moreover, the WIS method can also be used to determine the residual stress at the bore of the pipe.
On the other hand the following best fit polynomial equation was obtained for the 180mm length samples.

$$\frac{S_x}{S_{internal\ face}} = 0.937 * (\frac{x}{t})^2 - 2.9466 * (\frac{x}{t}) + 1.0142$$

**Equation 5**

$R^2$ =0.997

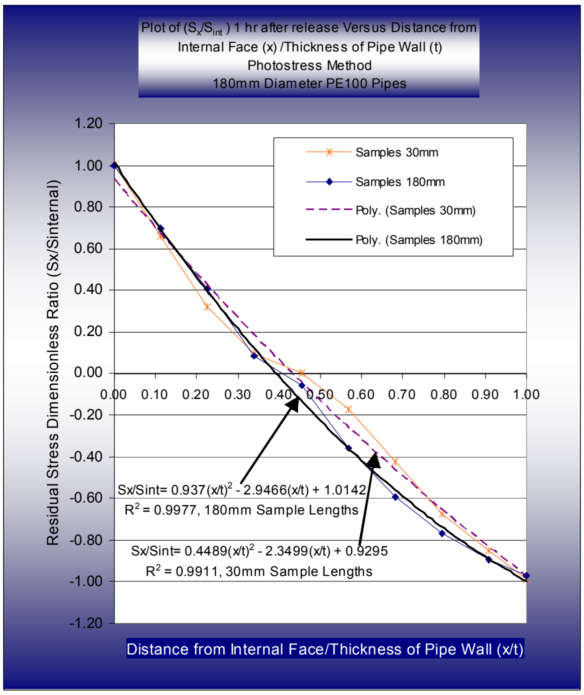


Figure 5: Dimensionless Plots of $S_x/S_{(Interior\ face)}$ Versus Distance $x$ from Internal Face/Thickness of Pipe t for 180mm and 30mm Length Samples.

### 3.1. Rigorous Idealised General Equation for Hoop Residual Stress Profile:

Following the above exercises of finding an ideal equation for plotting the hoop residual stress profile across the pipe wall it was concluded that the shape of the residual stress profile was not strictly replicated by the squared polynomial used in the Excel Trendline function. Hence, it was decided to carry out a more rigorous analysis to determine an idealised equation.
The rigorous analyses showed that the shape of the residual stress profile can be represented by applying the following equations:

$$\frac{x}{t} < z \qquad \frac{S_x}{S_{internal}} = \frac{\left|z - \frac{x}{t}\right|^n}{\left(z + (1 - 2z)\frac{x}{t}\right)^2}$$

**Equation 6**

$$\frac{x}{t} \geq z \qquad \frac{S_x}{S_{internal}} = 2\left(\frac{\frac{x}{t} - z}{z - 1}\right) + \frac{\left|z - \frac{x}{t}\right|^n}{\left(z + (1 - 2z)\frac{x}{t}\right)^2}$$

**Equation 7**

The notations $x, t, S_x, and\ S_{internal}$ are defined above whereas $n\ and\ z$ are coefficients to be solved for best fit graph.

For the 30mm sample lengths the following optimal coefficients in the above Equations 6 and 7 were calculated:
n=2 and z =0.527
Thus the idealised Equations for the **30mm sample** lengths were:

$$\frac{x}{t} < 0.527 \qquad \frac{S_x}{S_{internal}} = \frac{\left|0.527 - \frac{x}{t}\right|^2}{\left(0.527 - (0.054)\frac{x}{t}\right)^2}$$

**Equation 8**

$$\frac{x}{t} \geq 0.527 \qquad \frac{S_x}{S_{internal}} = \left(2.2283 - 4.2283\frac{x}{t}\right) + \frac{\left|0.527 - \frac{x}{t}\right|^2}{\left(0.527 - (0.054)\frac{x}{t}\right)^2}$$

**Equation 9**

$R^2 = 0.9926$

The n and z values for the **180mm sample** lengths were n= 2 and z=0.49 and the idealised equations were as follows:

$$\frac{x}{t} < 0.49 \qquad \frac{S_x}{S_{internal}} = \frac{\left|0.49 - \frac{x}{t}\right|^2}{\left(0.49 + (0.02)\frac{x}{t}\right)^2}$$

**Equation 10**

$$\frac{x}{t} \geq 0.49 \qquad \frac{S_x}{S_{internal}} = \left(1.9216 - 3.9216\frac{x}{t}\right) + \frac{\left|0.49 - \frac{x}{t}\right|^2}{\left(0.49 + (0.02)\frac{x}{t}\right)^2}$$

**Equation 11**

$R^2 = 0.9906$

Figure 6 shows the rigorous idealised plots both for the 30mm and the 180mm sample lengths. The residual stress profile follows an "Omega" shape form and follows closely the experimental data obtained for each of the 30mm and 180mm sample lengths. The R squared values calculated between the idealised equations and the actual observed data for each of the 30mm and 180mm samples were 0.9926 and 0.9906 respectively. These values demonstrate good closeness of fits for each sample length and shows that the rigorous idealised equations postulated mimic well the hoop residual stress profile in the pipe samples.

## 4. Conclusions:

- Figure 3 shows the results evaluated from the photostress observations made on three different samples HDPENS21, HDPENS22 and HDPENS23. These were consecutive samples of 180mm length cut from the same pipe. The desired reproduceability of the residual stress profile is remarkable as shown in Figure 3. The slight differences in magnitude are between 5 to 10% but the overall residual stress profile is consistent.
- Figure 4 and Equation 3 showed that the Photostress Method results from the 30mm length of samples can be used to determine the residual stress profile for an equivalent 180mm length of sample. This relationship facilitates the process of determining the residual stress profile and obviates the need to carry out tests with bulky 180mm length samples.

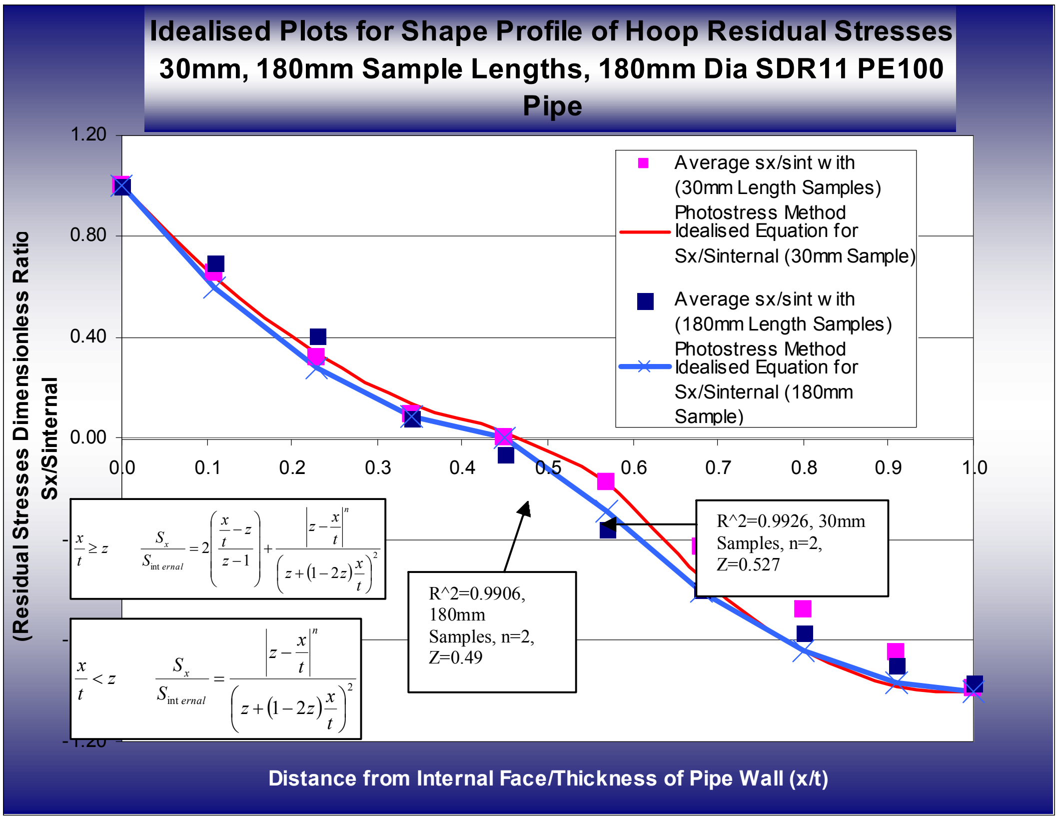


Figure 6: Dimensionless Rigorous Idealised Plots of $S_x/S_{(\text{Interior face})}$ Versus Distance $x$ from Internal Face/Thickness of Pipe t for 180mm and 30mm Length Samples.

- Further rigorous analyses were carried out to determine the shape of the residual stress profile as represented by Equations 8 and 9 for the 30mm length of pipe samples and Equations 10 and 11 for the 180mm (1x diameter) length of pipe samples. Figure 6 shows the plot of the shape profiles for both the 30mm and 180mm lengths of pipe samples. The equations showed that the shape of the residual stress profile mimics an "Omega" shape with a square polynomial.
- The plots from the experiments also showed that there were tensile stresses at the bore and compressive stresses at the external surface of the pipe confirming the findings from previous authors.
- The results showed that the Photostress Method is an effective method for determining the magnitude and profile of the residual stresses in HDPE pipes. The method requires lesser preparation time than the conventional Layer Removal Method and Subsequent Slitting Method (LRSS) proposed by authors such as Williams et al. (1981), Chaoui et al. (1988) and Isaac et al. (1995). The authors also postulated that due the intrusive nature of the LRSS method, the residual stress values obtained would have been modified stresses whereas the Photostress Method does not require any material removal to get the residual stress values at the inner part of the pipe wall thickness.

- The main corrections needed when using the Photostress Method are:
  (a) Corrections to take account of the stiffness of the coating in relation to the parent material. The coating should be chosen such that it does not reinforce the parent pipe material i.e use very low modulus coating compared to the parent test material,
  (b) Corrections due to the temperature variations during the tests which should be monitored closely and the readings taken should be corrected accordingly if needed, and
  (c) Parasitic fringes induced during the sample preparation and gluing process which can be eliminated by applying either a correction or by redoing the sample preparation.

## 5. Acknowledgements:

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